1. Suppose we have a collection of 1000 practice problems, 100 of the problems are considered challenging. Suppose that the final exam consists of 20 randomly chosen problems. Let $X$ be the random variable recording the number of challenging problems on the exam.

(a) What is the pdf of $X$?
(b) What is $E[X]$ and $V(X)$?
(c) What is the probability there are at least 2 challenging problems on the exam?

2. Prove the claim made in class that

$$V(aX + b) = a^2V(X)$$

where $a$ and $b$ are constants.

3. Suppose in a lottery you pick five different numbers from 1 to 90. Then five winning numbers are drawn. If you picked two of them, you win 20 dollars. For three, you win 150 dollars. For four, you win 5,000 dollars, and if all five match, you win a million dollars.

(a) What does Markov’s inequality predict about the probability that you will win at least 20 dollars?
(b) What does Chebyshev’s inequality predict about the probability that you will win at least 20 dollars?
(b) What is the actual probability that this happens?

4. Suppose you have a set of $n$ keys one of which opens the door to your new room. You pick a key equally likely at random and try the door. If it doesn’t work, you try again choosing only from the keys you haven’t tried before and repeat until you find the right key.

What is the variance of the number of keys you try until you open the correct door?

5. Suppose I play the following game: I flip a fair coin $n$ times, if it comes up Heads I win a dollar if it comes up tails I lose a dollar.
(a) What is the variance of my winnings?

Now suppose I flip a fair coin $n$ times, if it comes up Heads I win a dollar, if it comes up tails I simply win nothing.

(b) Bound the probability that my winnings differs from $\frac{n}{2}$ by more than $\sqrt{n}$. Give a numerical answer not in terms of $n$.

6. Suppose we have a deck of $n$ cards labeled 1 through $n$. I am going to perform a magic trick and ask you to select a card equally likely at random from the deck. Let $X$ be the random variable that records the number on the card you pick. Give closed form expressions for:

(a) $E[X]$
(b) $V(X)$