Pedestrian traffic models

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Introduction to Pedestrian Dynamics

Pedestrian dynamics

- two-dimensional nature
- should take into account interactions with other individuals that might cross walking path
- interactions may depend on the relative direction of the velocities

Models mostly work with the *operational level* of behavior, which deals with the actual walking behavior of pedestrians, including interactions with others and collision avoidance.

Observable quantities

Flow through a facility of width b is related to average density ρ and with average speed v of the pedestrian stream:

$$J = \rho v b = J_s b \,,$$

where the specific flow $J_s = \rho v$ is the flow per unit width (and is measured in $(m \times s)^{-1}$.)
The density is obtained from

$$\rho = \frac{N}{A} \,,$$

where N is the number of pedestrians within a selected area A.

Fundamental diagram

- Still controversial whether have to distinguish between unidirectional and bidirectional fundamental diagrams.

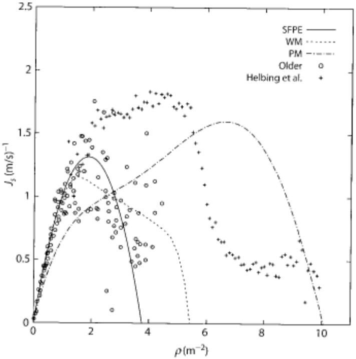


Figure 11.2 Fundamental diagrams for pedestrian movement in planar facilities. The lines refer to specifications according to planing guidelines (SFPE Handbook [1038], Predtechenskii and Milinskii (PM) [1138], Weidmann (WM) [1453]). Data points give the range of experimental measurements (Older [1080] and Helbing et al. [531]).

Types of models

Individual-based models

- Cellular automata
- Social force models
- Geometrical models
- Optimal Velocity models
- Lattice gas models

Macroscopic models

- Fluid dynamics analogy $\rightarrow$ LWR type models

Cellular automata models

- Define a cell size for a pedestrian to be an area of $40X40cm^2$.

Types of **neighborhoods**

- *Von Neumann neighborhood* of a cell: all cells that share an edge with it.
- *Moore neighborhood* of a cell: all cells that share a corner with it.

Factors that influence pedestrian motion:

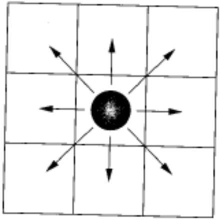
- Desired direction of motion toward the destination
- Interactions with other pedestrians: repulsive (short-distance), attractive (long-distance)
- Interactions with infrastructure

Floor field CA

- The standard cellular automata approach to pedestrian dynamics, able to reproduce collective effects.
- Inspiration from the process of chemotaxis.
- The purpose of this 'trace' is to transform effects of long-ranged interactions into a local interaction.
- Implemented in a two-dimensional stochastic cellular automaton: a *floor field model*.
- The **static floor field**: constant in time, shows constant properties of the infrastructure.
- The **dynamic floor field**: models dynamic interactions between pedestrians, but also has its own dynamics: diffusion and decay $\rightarrow$ corresponds to the trace.

Floor field CA

- Transition probabilities for moving to the neighboring cells are based on the floor fields.
- May also have a *matrix of preference*, which can encode the desired speed and direction of motion.



$p_{-1,-1}$	$p_{-1,0}$	$p_{-1,1}$
$p_{0,-1}$	$p_{0,0}$	$p_{0,1}$
$p_{1,-1}$	$p_{1,0}$	$p_{1,1}$

Figure 11.12 A particle, its possible directions of motion, and the corresponding transition probabilities p_{ij} for the case of a Moore neighborhood. For the von Neumann neighborhood, the diagonal probabilities $p_{\pm 1,\pm 1}$ are zero.

Update rules

Transition probabilities p_{ij} for movement to neighboring cell in direction (i,j) where $i,j \in \{-1,0,1\}$:

$$p_{ij} = N e^{k_D D_{ij}} e^{k_S S_{ij}} M_{ij} (1 - n_{ij}) \xi_{ij} \,,$$

where

$$\begin{aligned}
D &= \text{dynamic floor field matrix}\,, \\
S &= \text{static floor field matrix}\,, \\
M &= \text{matrix of preference for motion}\,, \\
N &= \text{normalization factor}\,, \\
n_{ij} &= \text{occupation number of the neighbor cell in the direction } (i,j)\,, \\
\xi_{ij} &= \text{geometry/obstacle number, 0 for forbidden cells (walls), 1 else}\,, \\
k_D, k_S &= \text{coupling strengths}\,.
\end{aligned}$$

Observations - Dynamic Floor Field

- Dynamic floor field is updated by the motion of the pedestrians, and also subject to diffusion and decay.
- When a particle moves from (x, y) to $(x + i, y + j)$, update $D_{xy} \to D_{xy} + 1$.
- D_{xy} has nonnegative integer values, which decay and diffuse to the neighboring cells with certain parameter probabilities.
- The rules have to be applied to all pedestrians at the same time - **parallel dynamics**.

Observations - Static Floor Field

Construction of static floor field

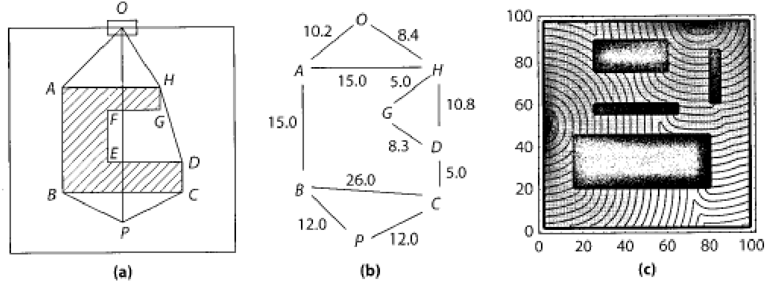


Figure 11.14 (a) Example for the calculation of the static floor field for a room with one obstacle. The door is at O, and the obstacle is represented by lines A–H. (b) The corresponding visibility graph for this room. Each node is connected by a bond if there are no obstacles between them. The real number on each bond represents the distance between nodes. (c) Contour plot of the static floor field for a room with four obstacles and two doors. The darkness of shading is inversely proportional to the distance from the nearest door.

Conflicts and friction

Conflict - occurs when more than one particle chooses the same cell destination.
Introduce a friction parameter μ: if 2 or more pedestrians want to move to the same target cell, all are denied with probability μ.
μ acts like a local pressure between the pedestrians.

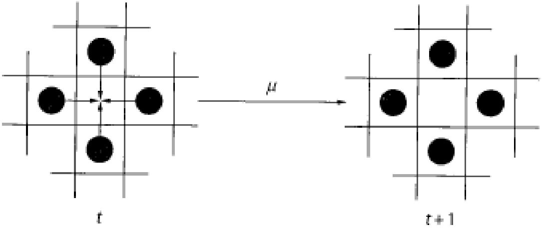


Figure 11.15 Refused movement due to the friction parameter μ (for $m = 4$).

Evacuation simulation

- For small k_S (coupling to static field), particles perform a random walk.
- For large k_S, particles will find the shortest possible path.
- At high densities, a large jam forms quickly at the exit.

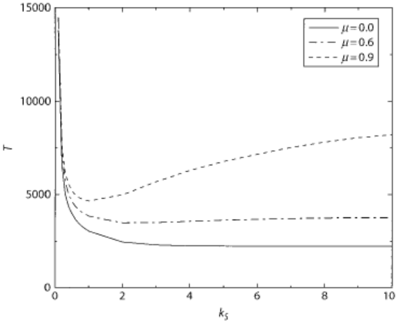


Figure 11.21 Dependence of evacuation times on the friction parameter μ and k_S for density $\rho = 0.3$.

Interactions with walls and other pedestrians

Interactions with walls - introduce an additional **wall potential**:

$$p_{ij}^{W} = e^{k_W \min(D_{\max}, d_{ij})} ,$$

with

$$\begin{aligned} k_W &= \text{sensitivity constant} , \\ D_{\max} &= \text{range of wall potential} , \\ d_{ij} &= \text{minimum distance of pedestrian from all walls} . \end{aligned}$$

Interactions with pedestrians - introduce a **politeness factor**:

$$p_{ij}^{P} = e^{-k_P N_p(i,j)} ,$$

where

$$N_P(i,j) = \text{number of pedestrians in the Moore neighborhood of } (i,j) .$$

Real-encoded CA

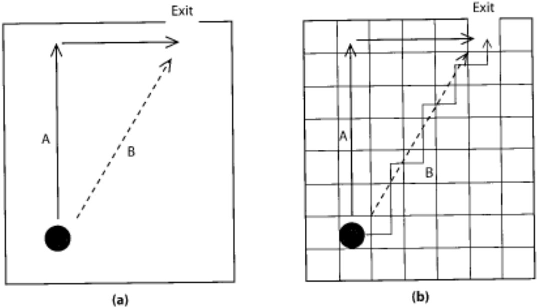


Figure 11.16 Example of an evacuation toward the exit, with two paths of A and B. (a) Movement in real situation without grid points. (b) Movement on a discrete CA lattice. Paths A and B have now the same length.

Options:

- Rescale probabilities to use Moore neighborhoods.
- Use real-coded cellular automata, where position and velocity of pedestrians are real numbers (need rounding procedure).

$V_{max} > 1$ - movement beyond nearest neighbors

- To reproduce asymmetry in the fundamental diagram, need $v_{max} > 1$.
- Neighborhood extended to all cells that can be reached in one time step.

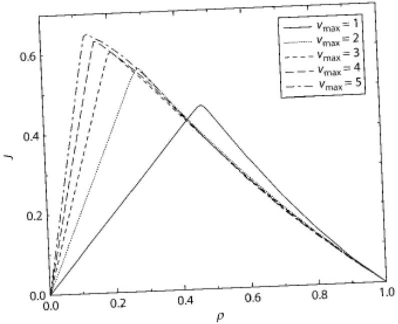


Figure 11.17 Fundamental diagrams of the floor field model for $v_{max} = 1, \ldots, 5$. The maximum is shifted toward smaller densities for increasing v_{max}.

Lane formation

Lane formation out of a randomly distributed group of pedestrians. Even and odd number of lanes may form.

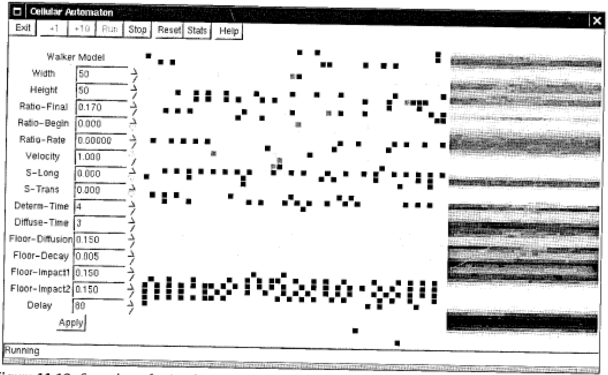


Figure 11.18 Snapshot of a simulation of counterflow along a corridor. The left part shows the parameter control. The central window is the corridor, and the light and dark squares are right- and left-moving pedestrians, respectively. The right part shows the dynamic floor fields for the two species.

Social force models

- Continuum model for pedestrian dynamics
- Interactions between pedestrians and influence from the environment encoded in a **social force/field**.

The basic equation of motion is

$$m_j \frac{d\mathbf{v}_j}{dt} = F_j^{(pers)} + F_j^{(soc)} + F_j^{(phys)},$$

where

$$\begin{aligned} m_j &= \text{mass of pedestrian j}\,, \\ \mathbf{v}_j &= \text{velocity of pedestrian j}\,, \\ F_j^{(pers)} &= \text{personal force}\,, \\ F_j^{(soc)} &= \text{total force due to other pedestrians}\,, \\ F_j^{(phys)} &= \text{physical force such as friction and compression}\,. \end{aligned}$$

Personal force

- Personal force - driving term

$$F_j^{(pers)} = \frac{m_j}{\tau_j} \left(\mathbf{v}_j^{(0)} - \mathbf{v}_j \right) \, ,$$

where

$$\begin{aligned} \tau_j &= \text{reaction or acceleration time} \, , \\ \mathbf{v}_j^{(0)} &= \text{preferred personal velocity} \, . \end{aligned}$$

Social force

- Social force - repulsive force between pedestrians
- Pedestrians: disks of radius R_j and with position $\mathbf{r}_j$

$$F_{jl}^{(soc)} = A_j e^{\frac{R_{jl} - r_{jl}}{\xi_j}} \mathbf{n}_{jl} \,,$$

$$\begin{aligned}
A_j &= \text{strength}\,, \\
\xi_j &= \text{range of the interactions}\,, \\
R_{jl} &= R_j + R_l \text{ sum of the disk radii}\,, \\
r_{jl} &= |\mathbf{r}_j - \mathbf{r}_l| \text{ distance between centers of mass}\,, \\
\mathbf{n}_{jl} &= \frac{\mathbf{r}_j - \mathbf{r}_l}{r_{jl}} \text{ normalized vector pointing from l to j}\,.
\end{aligned}$$

- May add anisotropy factor $\lambda_j + (1 - \lambda_j)\frac{1+\cos\varphi_{jl}}{2}$.

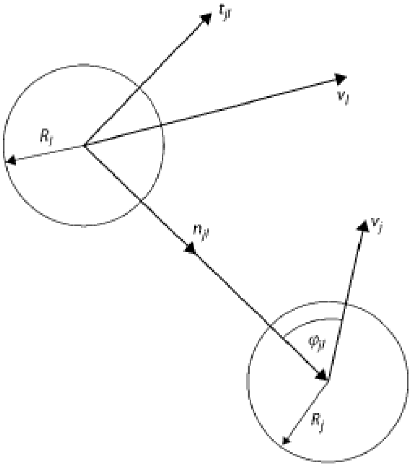


Figure 11.22 Definition of the most important quantities in the social-force model.

Physical force

- When $r_{jl} \leq R_{jl}$ (contact between pedestrians), physical force becomes relevant.
- It has two contributions:

$$F_{jl}^{(push)} = \kappa\Theta(R_{jl} - r_{jl})\mathbf{n}_{jl}\,,$$
$$F_{jl}^{(fric)} = \kappa\Theta(R_{jl} - r_{jl})\mathbf{t}_{jl}\,,$$

where

$$F_{jl}^{(push)} = \text{force that tries to prevent compression}\,,$$
$$F_{jl}^{(fric)} = \text{sliding friction force in tangential direction } \mathbf{t}_{jl}\,,$$
$$\kappa = \text{can be chosen as a function of the tangential velocity}\,,$$
$$\Theta = \text{Heaviside step function}\,.$$

Table 11.2 Typical parameter values for the social-force model.

Parameter	Value
$\mathbf{v}_j^{(0)}$	$0.6 - 1.5\,\mathrm{m/s}$
k	$1.1–1.2 \cdot 10^5\,\mathrm{kg/s^2}$
A_j	$2 \cdot 10^3\,\mathrm{N}$
ξ_j	$0.08\,\mathrm{m}$
τ_j	$0.5\,\mathrm{s}$
κ_j	$2.4 \cdot 10^5\,\mathrm{kg/m/s}$
R_j	$\in [0.25\,\mathrm{m}, 0.35\,\mathrm{m}]$

- Limitations: equations of motion describe particles with inertia.
- Can lead to violations of volume exclusion, or oscillations that lead to velocities in the opposite direction.

A class of nonlocal models for pedestrian traffic (Colombo et al)

Starting point: Cauchy problem for the conservation law

$$\partial_t \rho + div(\rho v(\rho)(\nu(x) + I(\rho))) = 0\,,$$
$$\rho(0,x) = \rho_0(x)\,,$$

where

$$\begin{aligned} \rho(x,t) &= \text{density of the moving crowd}\,,\\ v(\rho(x,t)) &= \text{speed of the pedestrian at time } t \text{ and position } x\,,\\ \nu(x) &= \text{preferred direction of the pedestrian at } x\,,\\ I(\rho(t))(x) &= \text{how the pedestrian at } x \text{ deviates from preferred direction,}\\ &\quad\ \text{given crowd distribution is } \rho\,. \end{aligned}$$

First choice of I

- Assume each pedestrian avoids high crowd densities.
- Fix mollifier η.
- $\rho * \eta$ is an average of crowd density around x.

Then take

$$I(\rho) = -\epsilon \frac{\nabla(\rho * \eta)}{\sqrt{1 + ||\nabla(\rho * \eta)||^2}}$$

- This leads to pattern formation (self-organization into lanes).
- Can also test Braess-like paradox: careful positioning of suitable obstacles near an exit might improve the outflow through the exit.

Second choice of I

- Assume there are restrictions on the angle of vision of each pedestrian.
- Fix mollifier η.
- Smooth function φ weights the deviation from the geodesic $g(x)$.

Then take

$$I(\rho) = -\epsilon \frac{\nabla \int_{\mathbf{R}^2} \rho(y)\eta(x-y)\varphi((y-x)\cdot g(x))dy}{\sqrt{1 + ||\nabla \int_{\mathbf{R}^2} \rho(y)\eta(x-y)\varphi((y-x)\cdot g(x))dy||^2}}$$

Second choice of I

Re-write the deviation

$$I(\rho) = -\epsilon \frac{\nabla \int_{\mathbf{R}^2} \rho(y)\eta(x-y)\varphi(x,y)dy}{\sqrt{1 + ||\nabla \int_{\mathbf{R}^2} \rho(y)\eta(x-y)\varphi(x,y)dy||^2}}$$

with possible choices

$$\varphi(x,y) = \hat{\varphi}((y-x)\cdot g(x)), \text{ or}$$
$$\varphi(x,y) = \hat{\varphi}\left(\frac{(y-x)\cdot g(x)}{\sqrt{1+||y-x||^2}}\right),$$

with $\hat{\varphi} \in \mathbf{C}^3(\mathbf{R}, [0,1])$ and

$$\hat{\varphi}(\xi) = \begin{cases} 0 & \text{if } \xi < 0 \\ 1 & \text{if } \xi > \mathscr{V} \end{cases}$$

Cone of visibility

$\mathscr{V} > 0$: related to the width of the cone of visibility for each individual

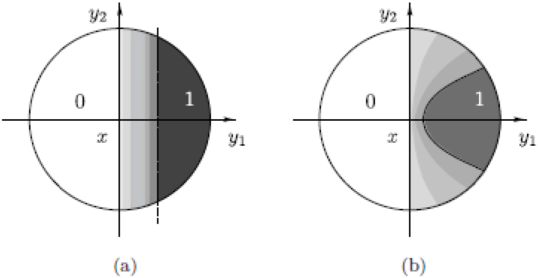


Fig. 1. The dark shaded region is the area where $\varphi(x, y) = 1$ inside the circle that describes the support of η, in the case $g(x)$ is the unit horizontal vector and x is at the origin. (a) refers to the choice on the left of (3.7), here ϑ is related to the position of the vertical line and (b) is related to the right choice in (3.7) and here ϑ is related to the position of the vertex of the shaded area.

Numerical simulations

- Choose $\nu = \nu(x)$ the geodesic one (shortest path)
- Use Lax-Friedrichs methods with dimensional splitting.

Consider first choice of I with:

$$\begin{aligned} \nu(x) &= \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \delta(x), \eta(x) = \left(1 - \left(\frac{x_1}{r}\right)^2\right)^3 \left(1 - \left(\frac{x_2}{r}\right)^2\right)^3 \chi_{[-r,r]^2}(x) \\ v(\rho) &= \frac{1}{2}(1-\rho), \rho_0(x) = \chi_{[3/5,4]\times[-3/5,3/5]}(x)\,, \\ r &= \frac{4}{5}, \epsilon = \frac{2}{5}\,. \end{aligned}$$

Lane formation

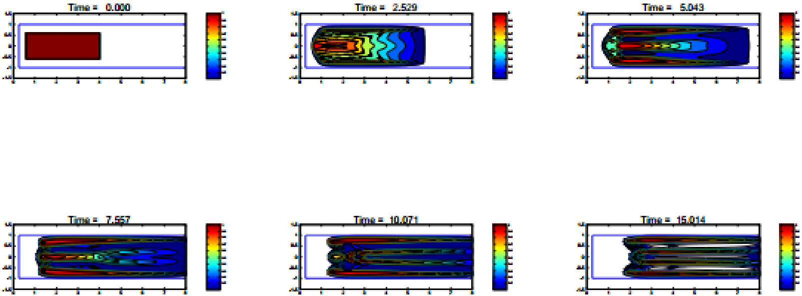


Figure 1: Solution to (1.1)–(1.2)–(3.4)–(4.1) at times $t = 0, 2.529, 5.043, 7.557, 10.071, 15.014$. First 3 lanes are formed, then the middle lane bifurcates forming the fourth lane.

Lane formation - parameter r

Parameter r: determines size of sppt of η, and leads to patterns with different numbers of lanes.

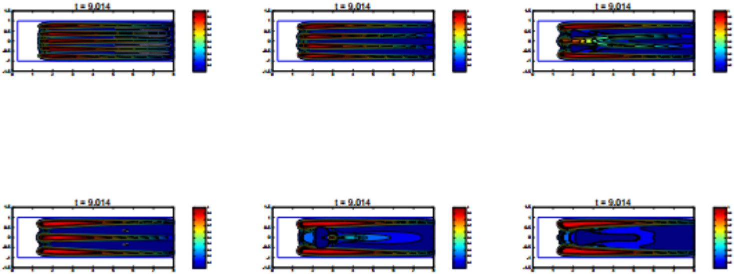


Figure 2: Solution to (1.1)–(1.2)–(3.4)–(4.1) computed at time $t = 9.014$ and with spt η with radius $r = 0.5, 0.6, 0.8, 0.9, 1.0, 1.4$. Note that as r increases, the number of lanes diminishes.

Lane formation - stable phenomenon

Different initial data leads to similar lane formation.

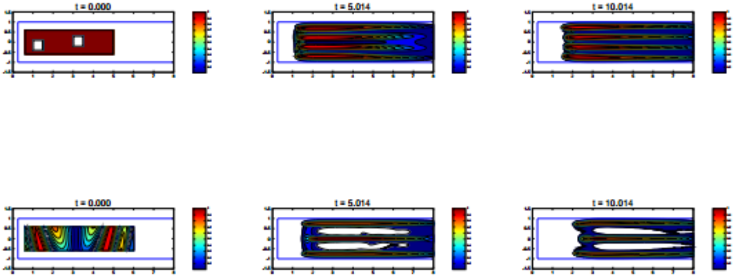
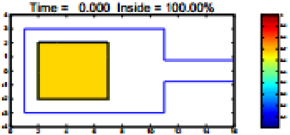


Figure 3: Solution to (1.1)–(1.2)–(3.4)–(4.1) with different initial data at time t = 0, 5.014, 10.014 and above with r = 0.6, below with r = 0.9. Note that above 4 lanes form and below 5, similarly to what obtained in Figure 1.

Numerical simulations - Room evacuation

- Standard application: minimization of exit times.
- Consider a room with an exit, and $\nu = \nu(x)$ is the unit vector tangent at x to the geodesic connecting x to the exit.



$$
\begin{aligned}
v(\rho) &= 6(1-\rho) \\
\eta(x) &= \left[1 - \left(\tfrac{x_1}{r}\right)^2\right]^3 \left[1 - \left(\tfrac{x_2}{r}\right)^2\right]^3 \chi_{[-r,r]^2}(x), \\
r &= 0.6 \\
\rho_0(x) &= 0.75\, \chi_{[2,7]\times[-2,2]}(x) \\
\varepsilon &= 0.4
\end{aligned}
$$

Room evacuation

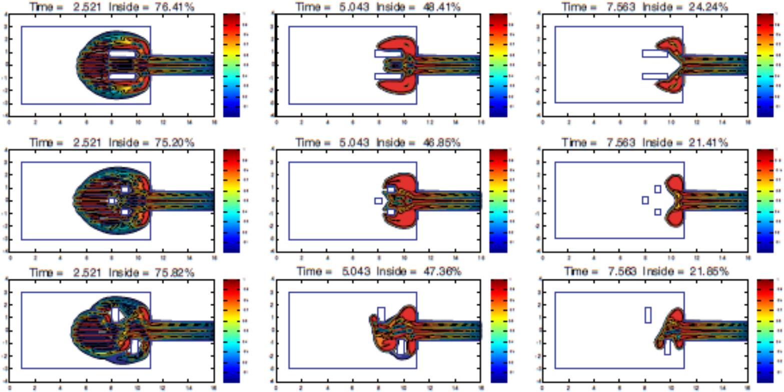


Fig. 6. Solution to (1.1), (1.2), (3.4) and (4.2) with different geometries, computed at time $t = 2.521$, 5.043 and 7.563.

Room evacuation

Careful introduction of suitable obstacles in suitable locations may reduce the exit time.

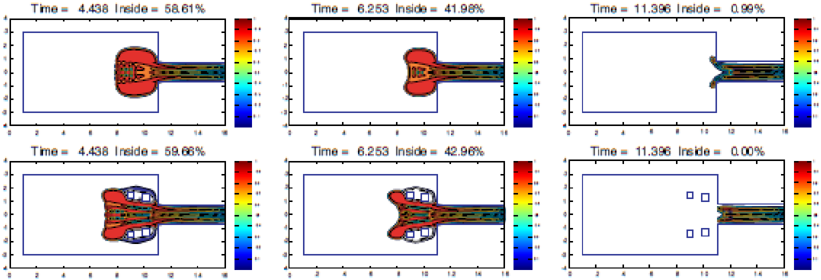


Fig. 7. Solution to (1.1), (1.2), (3.4) and (4.2) with $\varepsilon = 0.2$, at times $t = 4.438, 6.253, 11.396$. On the first line, no obstacle is present. On the second line, four columns direct the crowd flow. Note that the exit time in the latter case is *shorter* than in the former one.

References

A. Schadschneider, D. Chowdhury, K. Nishinari
Stochastic transport in complex systems: from molecules to vehicles - Chapter 11: Pedestrian Dynamics.
Elsevier, 2011.

R. Colombo, M. Garavello, M. Lecureux-Mercier.
A class of nonlocal models for pedestrian traffic
Math. Models Methods Appl. Sci. 22(04): 1150023 , 2012.

C. Rolland, P. degondo, S. Motsch.
Two-way multi-lane traffic model for pedestrians in corridors
Networks and Heterogeneous Media 6(3):351-381, 2011.

Thank you for your attention!