Homework 7

due Tuesday, July 21st at 1 pm

Submit your homework in class or in box #9 in 182 George St main lobby (on the left of the stairs going up) or hand it to one of the instructors. You can type or write your answers by hand. You are allowed to discuss the problems but you should write your own answers. Late homework is not accepted, except for medical reasons or similar emergencies.

Problem 1

In this problem, you will analyze a model of disease spread in a population (the SIR model). These equations model the spread of an endemic disease that confers immunity. The variables in the model represent individuals that are susceptible to the disease (S), infected by it (I), or recovered/removed (R). $N = S + I + R$ is the total number of individuals in the population.

The equations are as follows:

$$\frac{dS}{dt} = \mu N - \beta SI - \mu S \tag{1}$$

$$\frac{dI}{dt} = \beta SI - (\gamma + \mu)I \tag{2}$$

$$\frac{dR}{dt} = \gamma I - \mu R\,, \tag{3}$$

where $\mu > 0$ is the population birth and death rate, $\beta > 0$ is the transmission rate and $\gamma > 0$ is the rate of recovery from the infection.

(a) Calculate $\frac{dN}{dt}$ and conclude how the total population varies with time.

(b) Let $s = \frac{S}{N}$ and $f = \beta I$. Show that the differential equations in s and f become:

$$\frac{ds}{dt} = \mu(1 - s) - sf \tag{4}$$

$$\frac{df}{dt} = (\gamma + \mu)f(R_0 s - 1)\,, \tag{5}$$

where $R_0 = \frac{\beta N}{\gamma + \mu}$ is the "basic reproductive rate" of the disease. Note that s is the fraction of susceptibles in the population, and f is a measure of the force of the disease.

(c) Find the critical points of the system found in part (b). Your answer may depend on the parameter R_0.

(d) What does each critical point mean in the context of the disease?

(e) Determine the stability of the critical points.

(f) What do you conclude about the effect of R_0 on disease spread as predicted by this model?

Problem 2

Download the article "Ross_and_malaria" from Canvas (under *Files/Homework*). Read the article and provide 2-3 paragraphs with a brief summary of what you learned from this paper. Your summary should include the answers to these questions:

- Why did Ronald Ross decide to use mathematical models in his work?
- What differential equations did he use to model his system?
- What conclusion did he draw from the analysis of his differential equation system?
- Why were his qualitative results more important than the quantitative ones?