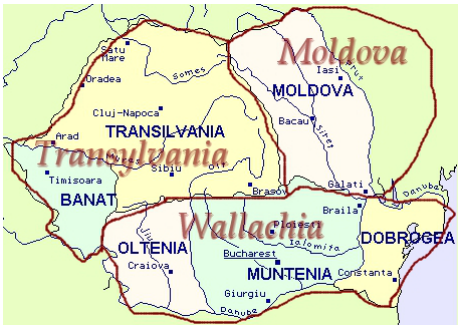
Worksheet: Applications of Graphs

1 Map coloring

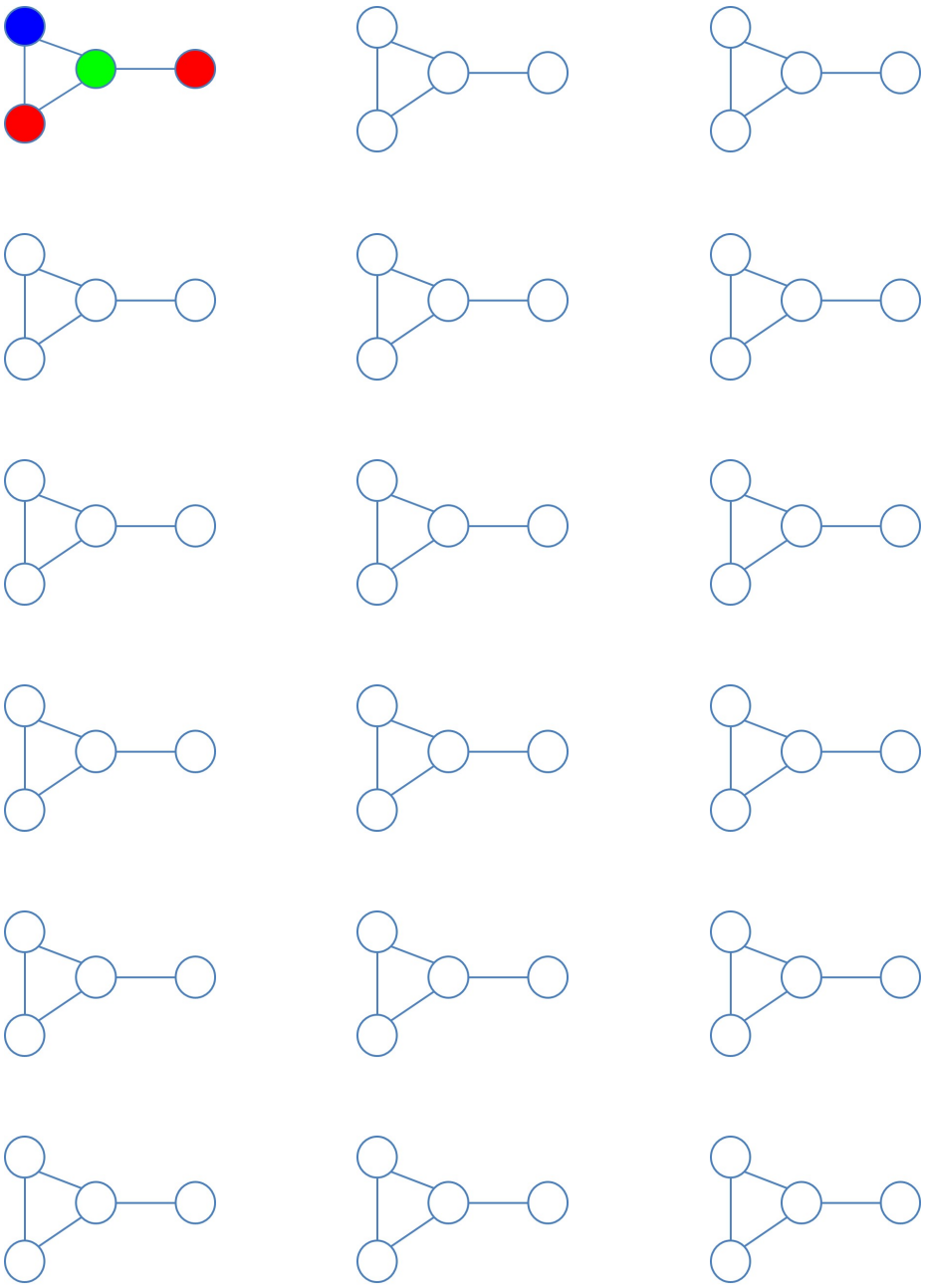
1.1

Consider the following maps. How can you form a graph based on these maps? What will the vertices and edges be in this representation? Draw the graphs right below the corresponding map.



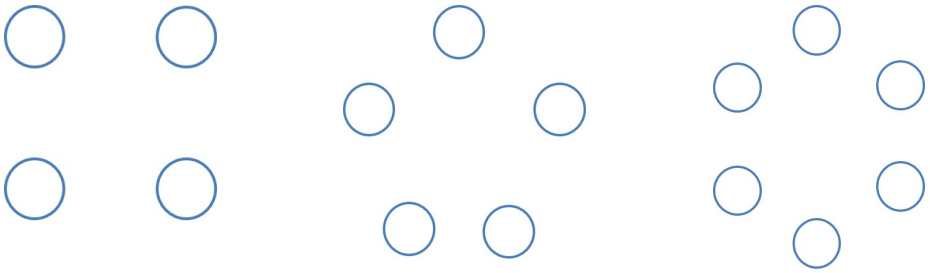
1.2

Draw all possible three-colorings of the graph below: the first is provided. How many such colorings can you find?



1.3

Draw a complete graph using the 4, 5 and respectively 6 vertices drawn below. Then try to create a coloring of each graph so that no vertices connected by an edge have the same color. How many colors do you need in each case?



What do you conclude about the chromatic number of a complete graph with n vertices?

2 Clustering and Epidemic Spread

2.1

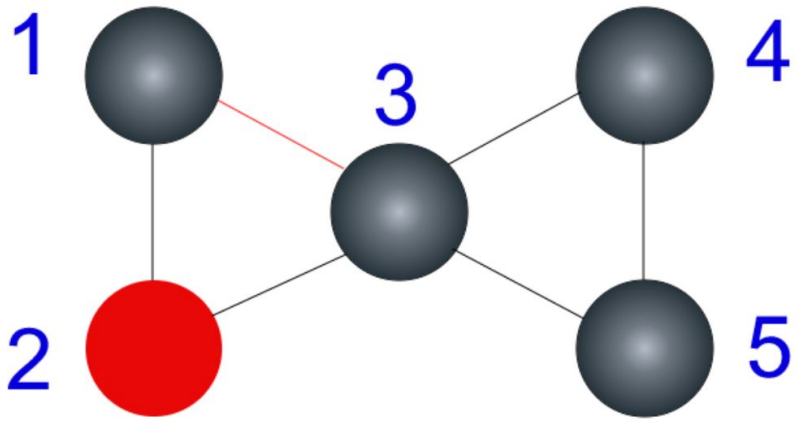
Now we consider the spread of an epidemic or of a rumor on a network. Fill out the table below to indicate what the vertices and edges in the graph could represent.

Graph/ network	Epidemic spread	Rumor/news spread
Vertices		
Edges		

Discuss and compare with people around you.

2.2

In this slide we calculated the clustering coefficient of node 1. It's your turn to calculate it for node 2!

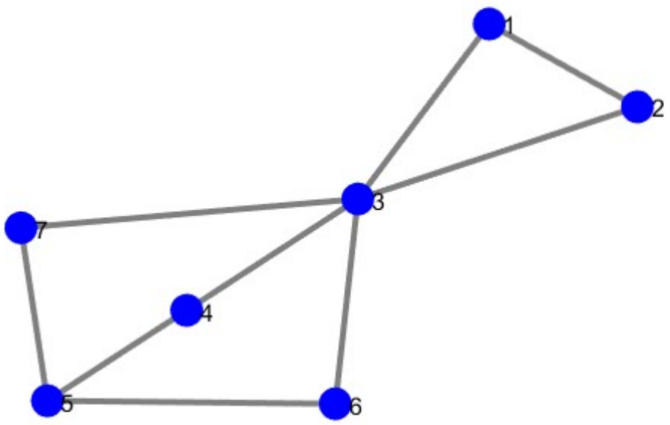


- First, calculate the degree of node 2: d_2.

- Next, calculate the clustering coefficient of node 2 using the formula in Definitions (last page of the Worksheet) and the slide example.

2.3

Challenge problem: If you have time left during any of the exercises, try to calculate the average clustering coefficient of the example network in the lecture.



Note that you first have to calculate the clustering coefficient of each node, and then average all these coefficients for your final answer. Check your answer with one of the TAs!

Definitions and formulas

Chromatic number:
The minimum number of colors required to color a map so that no adjacent regions have the same color.

Clustering:
Clustering is a measure of how clique-ish a network is. In other words, clustering measures the degree to which nodes in a graph tend to cluster (be linked) together.

Clustering coefficient of node i:

$$cl_i = \frac{\text{number of pairs of neighbors connected by edges}}{d_i * (d_i - 1)/2},$$

where $d_i =$ the degree of node i.

Average clustering coefficient of a network with n nodes:

$$\bar{cl} = \frac{cl_1 + cl_2 + ... + cl_n}{n}.$$

Degree centrality: The degree centrality of node i measures how influential/central individual i is to the network. It is calculated as follows:

$$c_i^d = \frac{\text{degree of node } i}{\text{sum of all node degrees}}.$$

Recall that degree centrality is a local measure that depends on the number of neighbors of each node.