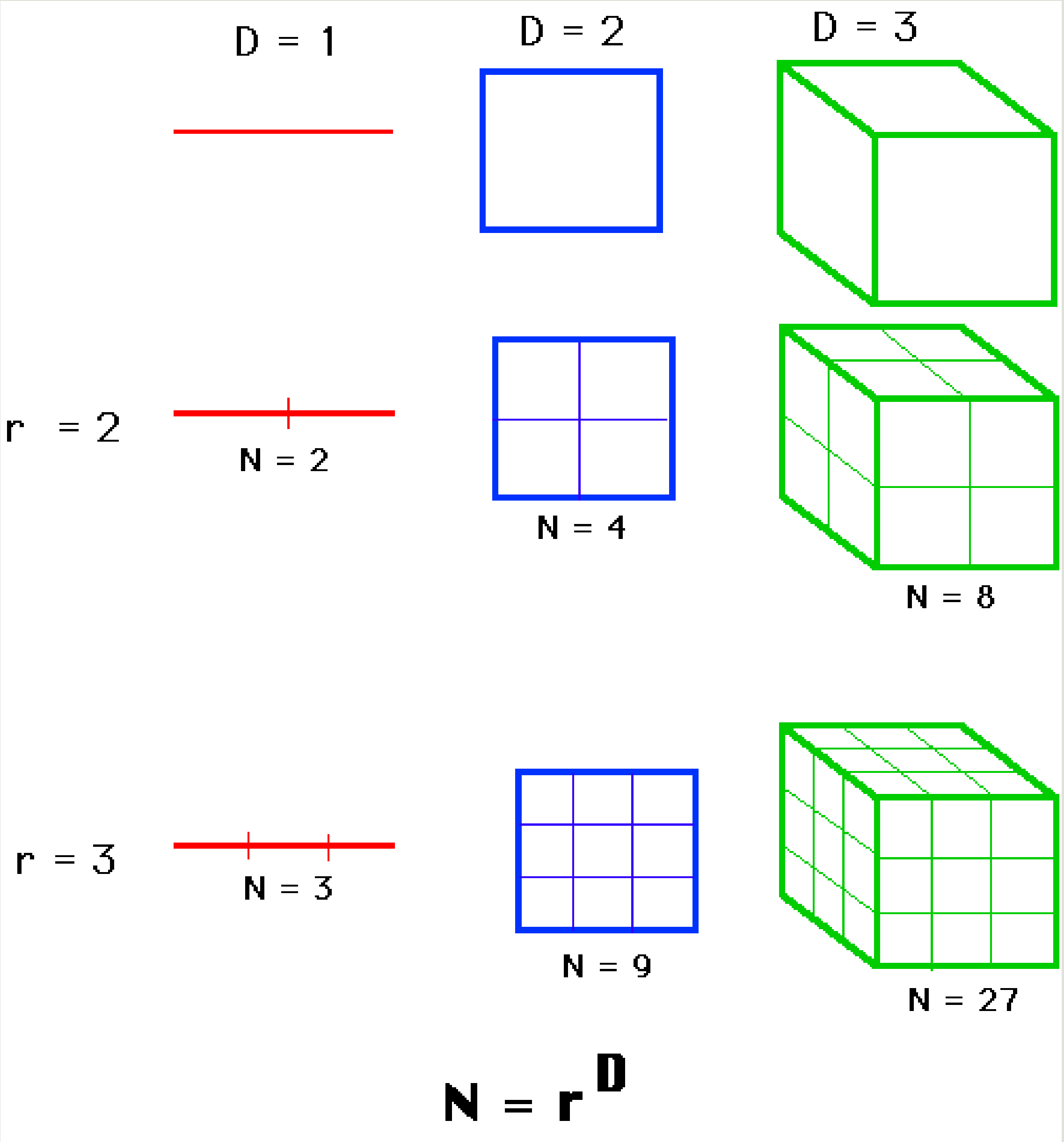
Fractals: properties and applications

MATH CO-OP

VERONICA CIOCANEL, BROWN UNIVERSITY

Fractal ball experiment: DIY!

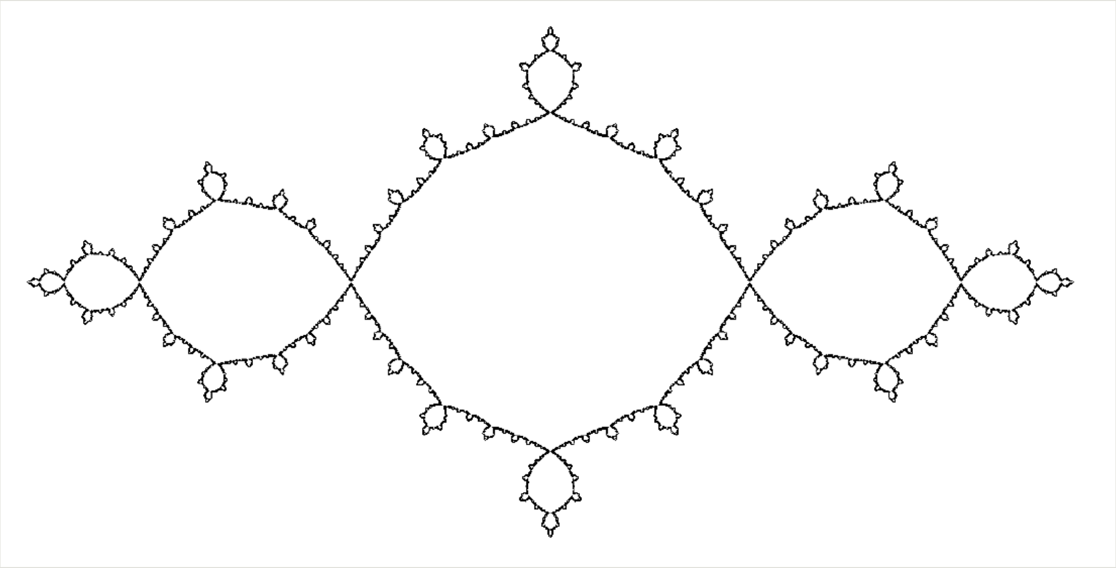
How do we think of dimension?



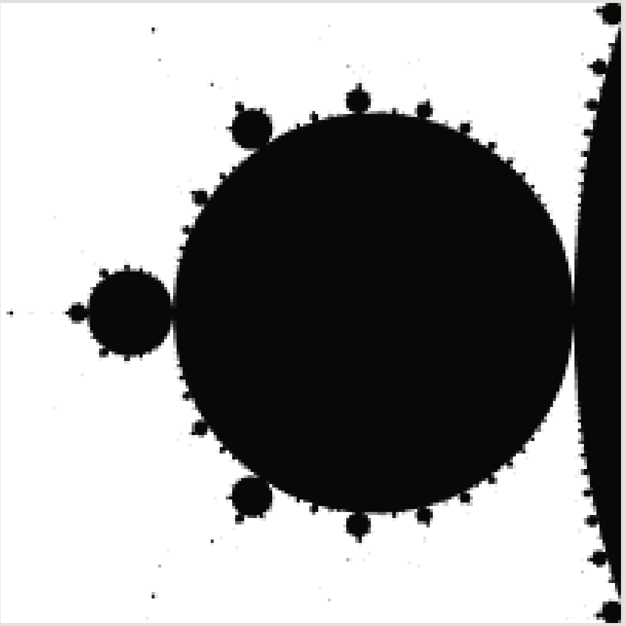
Conclusions: Fractal properties

- Fractals exhibit **fractal dimensions**: all objects whose dimension is not an integer are fractals.

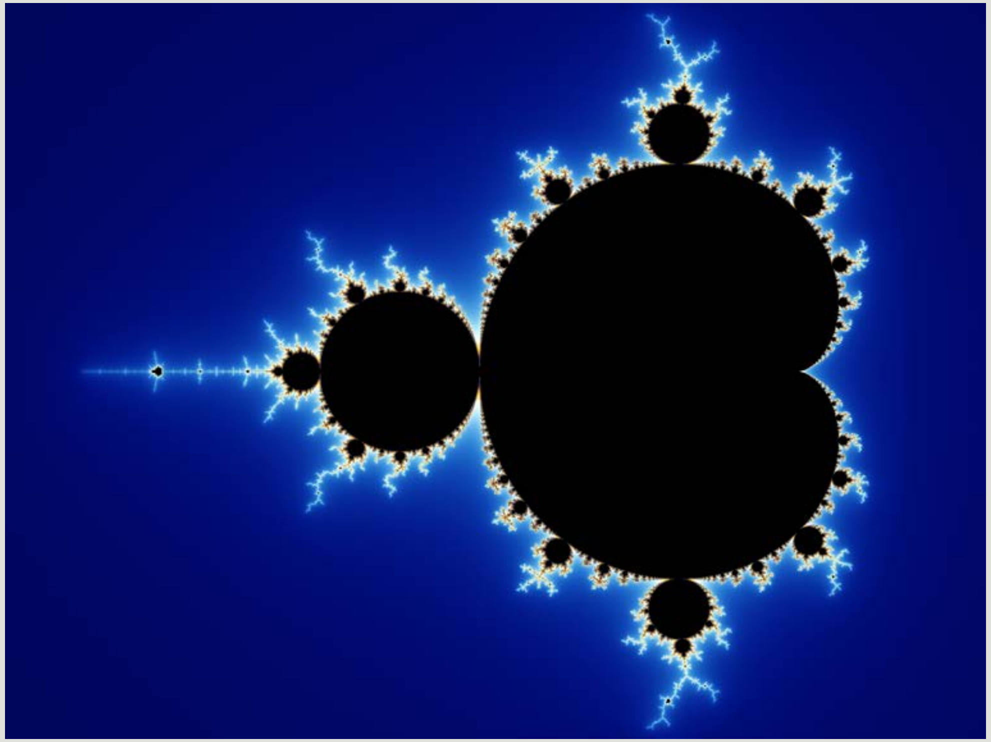
- Fractals are **self-similar**.



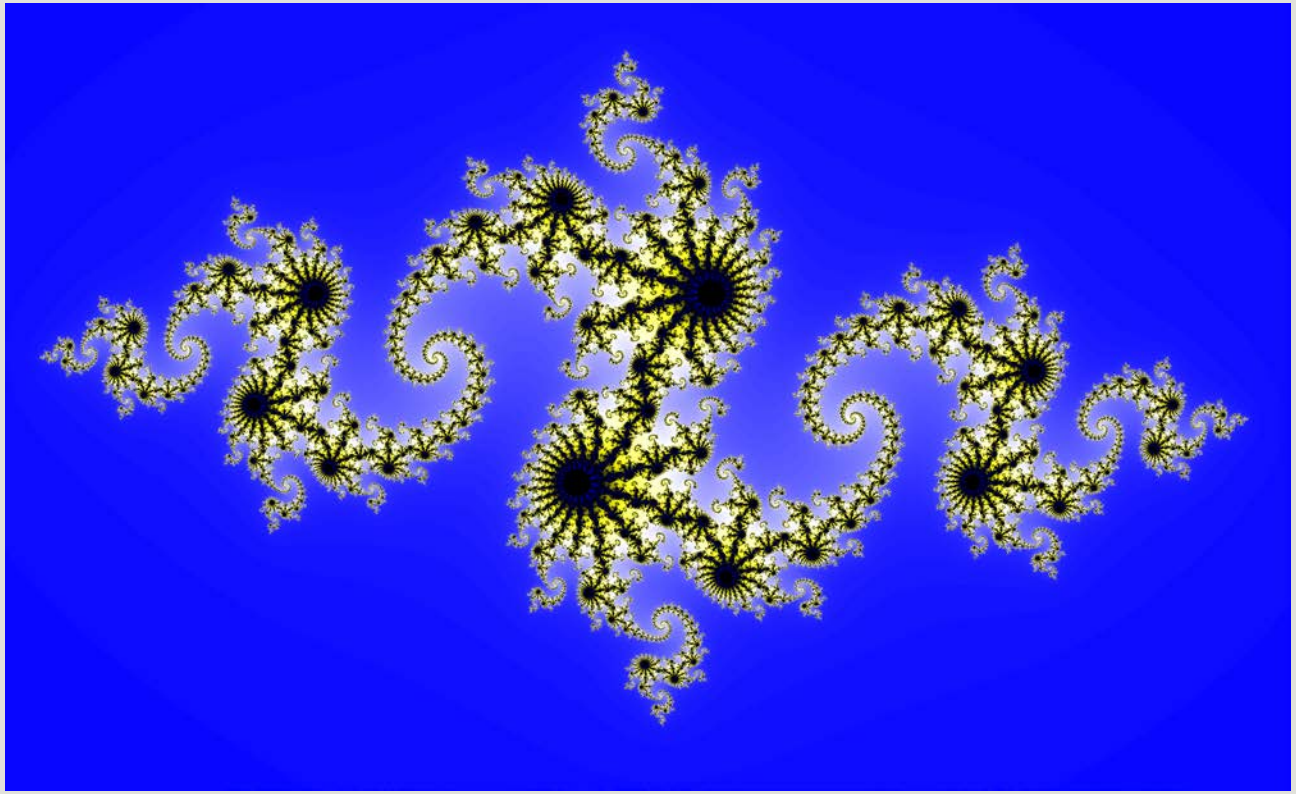
d = 1.2683



Fractals - mathematical objects

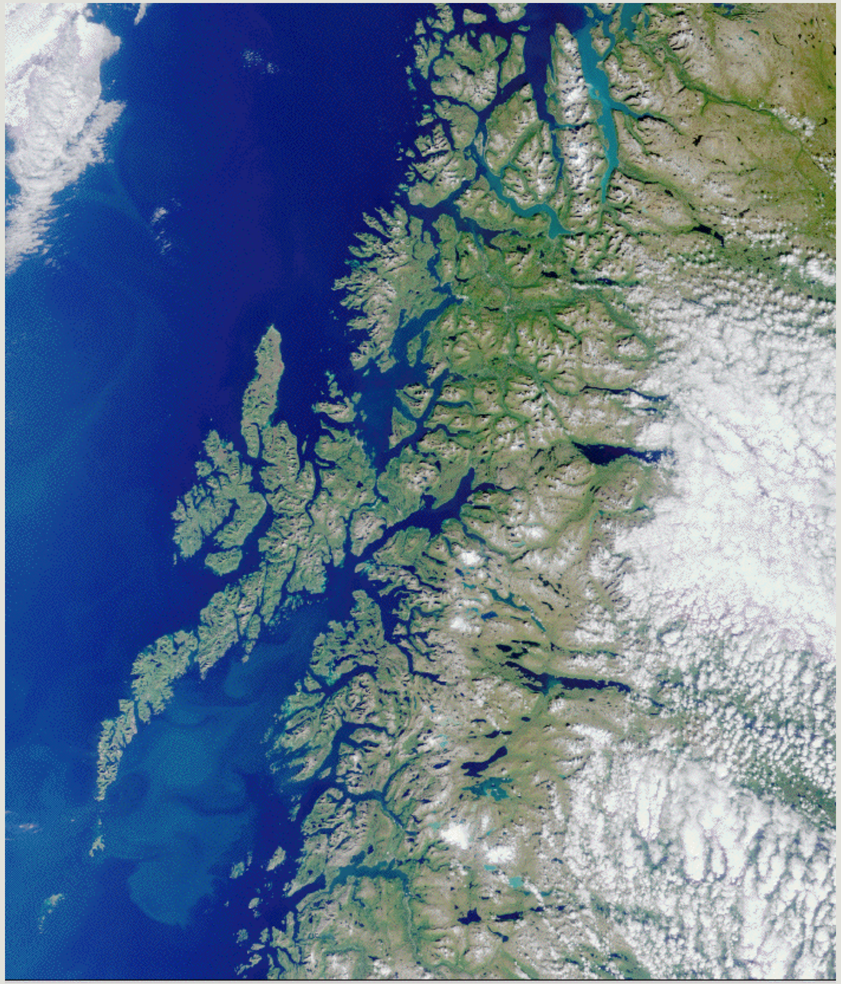


Mandelbrot set



Variation of a Mandelbrot set

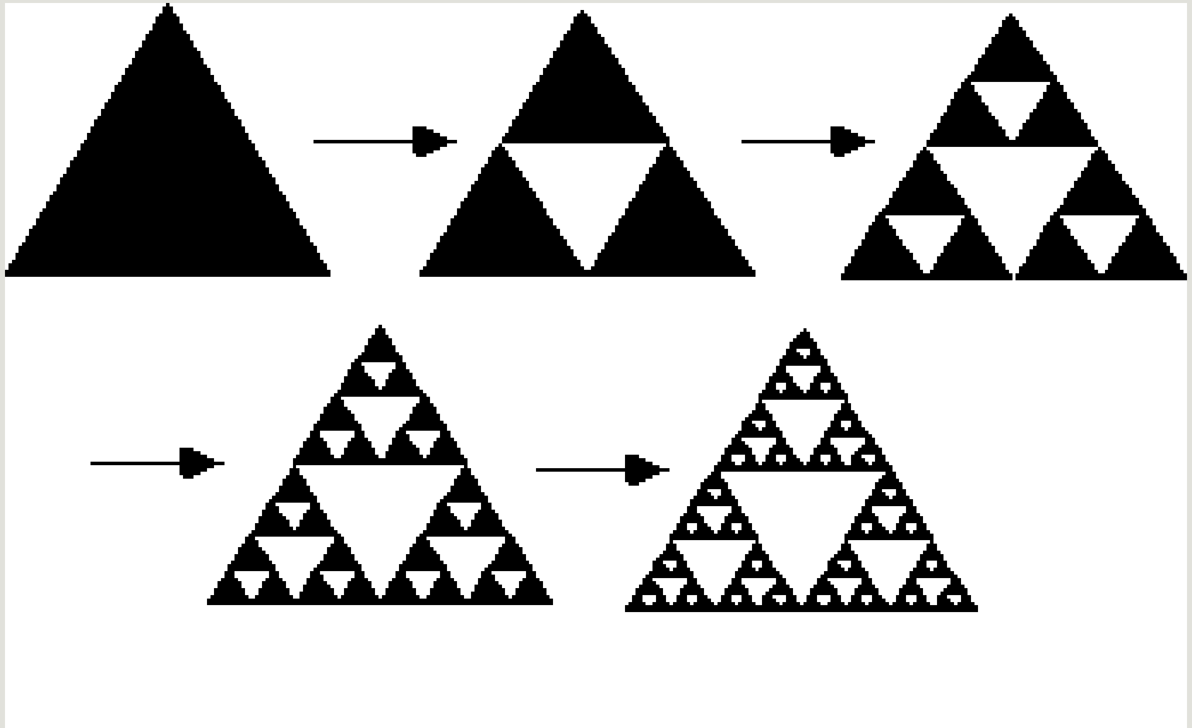
Fractals - around us



Lake Mead coastline

The Great Wave off Kanagawa - Hokusai

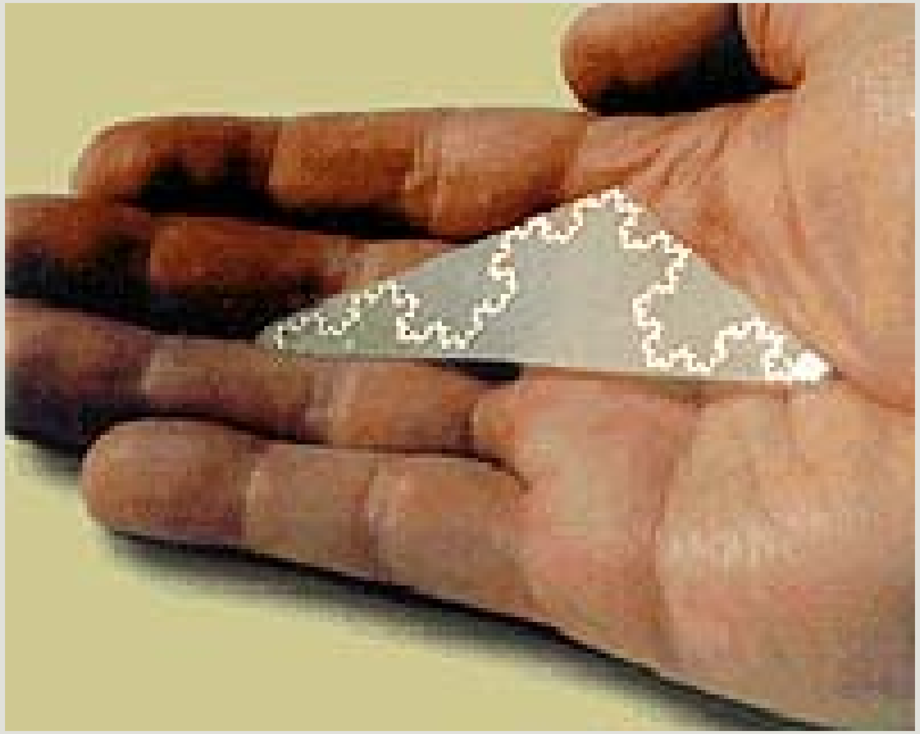
1. Fractal antennas



Sierpinski
triangle

Example of fractal
antenna

- Fractal-shape antennas can respond to more frequencies than regular ones.
- They can be ¼ the size of the regular ones: use in cellular phones and military communication hardware.
- BUT: Not all fractal shapes are best suited for antennas.



Koch curve fractal antenna

2. Coastlines

Border length

- Portugal - Spain border → 987 km (reported by the Portuguese)
- → 1214 km (reported by the Spanish)

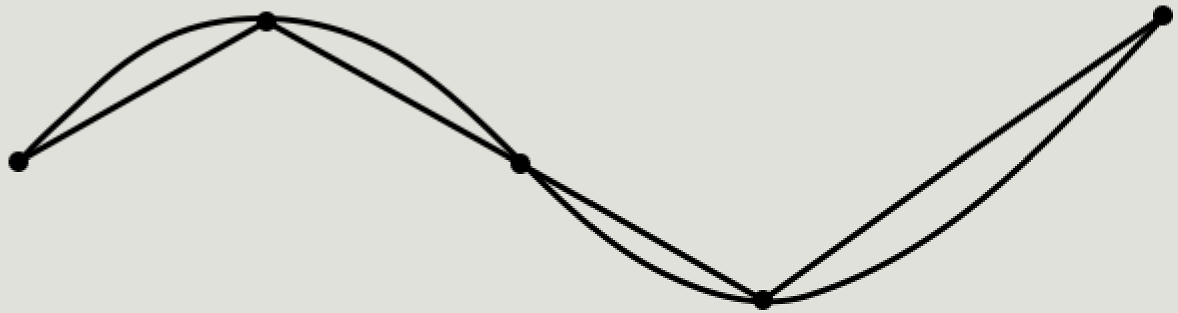
Measurements were using different scales!

Returning to coastlines…

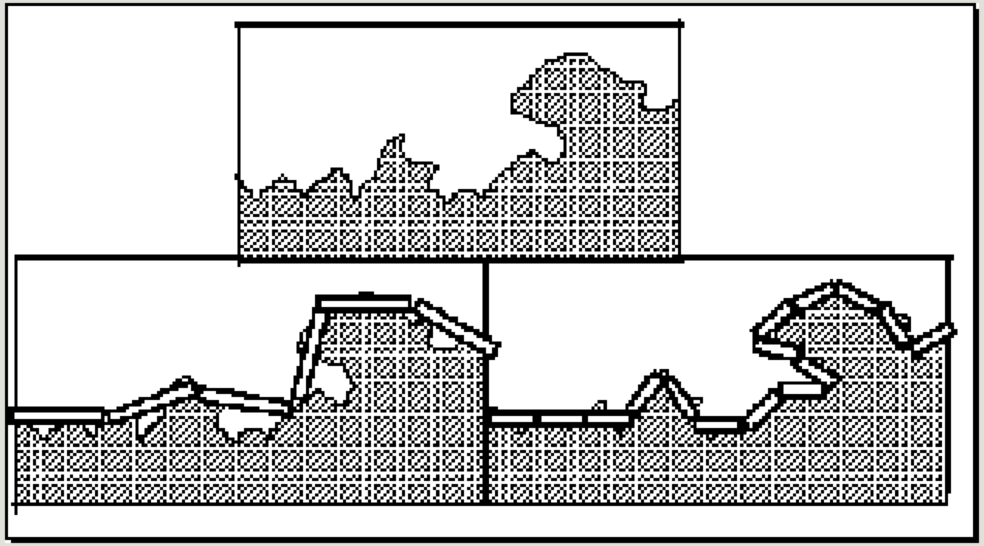
South Africa

Britain

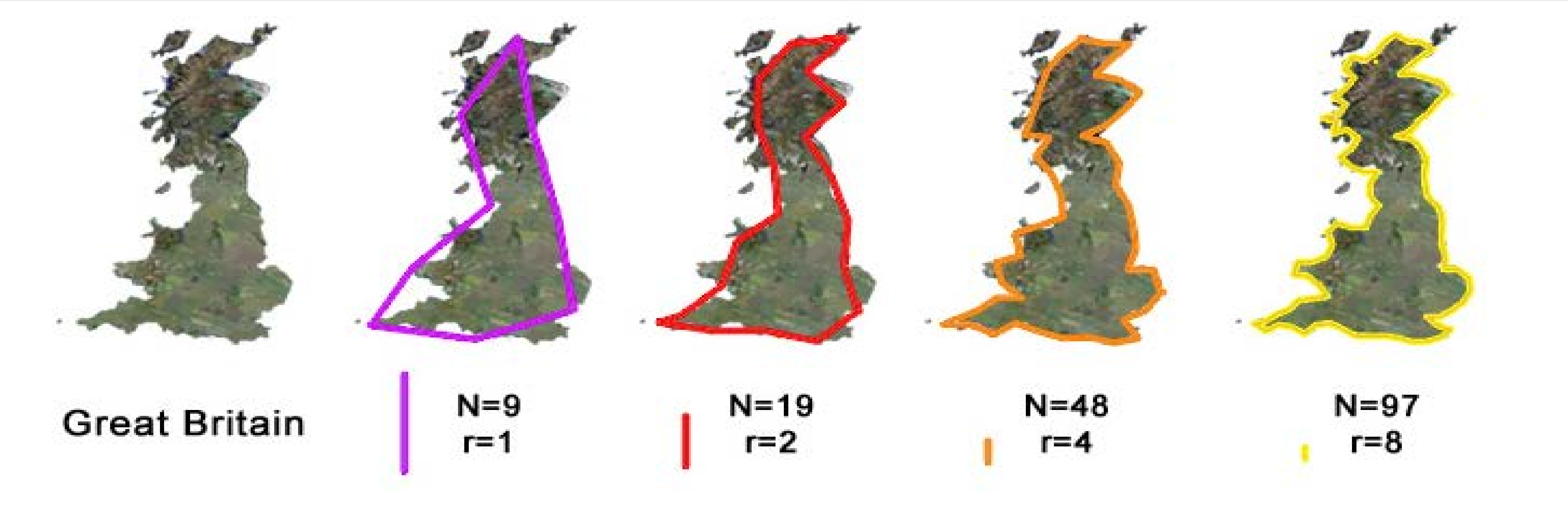
Approximating a smooth curve using straight lines – guaranteed to get closer to the true value of the curve length



Can we say the same for the UK coastline?



Scale/ruler length: $l = \frac{1}{r}$



Perimeter/length: $N * l$

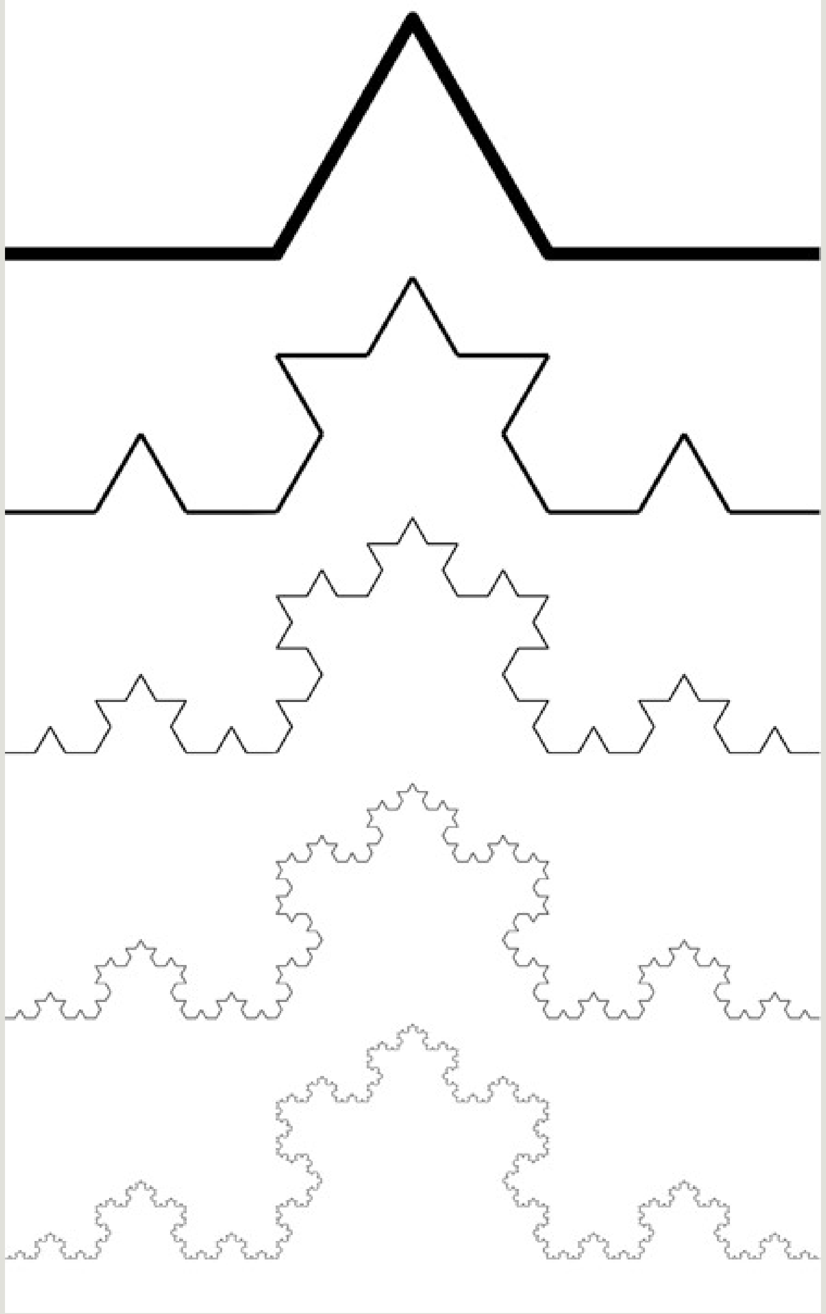
$9 * \frac{1}{1} = \mathbf{9}$ $19 * \frac{1}{2} = \mathbf{9.5}$ $48 * \frac{1}{4} = \mathbf{12}$ $97 * \frac{1}{8} = \mathbf{12.125}$

- Coastlines have fractal-like properties: complexity changes with measurement scale

- A lot like the Koch curve

- This curve has **infinite length**!

- **Length**: makes little sense



But, concept of **fractal dimension** makes sense!

South Africa: d = 1.02

Britain: d = 1.25

- This is called the **“Coastline paradox”**: measured length of a stretch of coastline depends on the measurement scale

- But for practical use, the ruler scale is not that fine: km’s are enough!

- Approximating the coastline with an infinite fractal is thus not so useful in this case.