Inverse Square Law

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1 What is the Inverse Square Law?

$$F = K\frac{qq_0}{r^2} \quad , \quad F = G\frac{m_1 m_2}{r^2} \quad , \ldots$$

The inverse square law is something that is ubiquitous among so many laws of physics, at least that is what I think. Although my exposure to physcis is limited, it seems to me that the inverse square law is fundamental to laws governing the forces between electric charges, all the way up to the forces between large bodies (i.e. gravitation). So why is it that the forces between certain bodies are inversely proportional to the square of the distance? For instance, why is it that if I bring an electron closer to another electric by half the distance, the force repelling the two will increase by an order of 4?

Let us consider the following picture

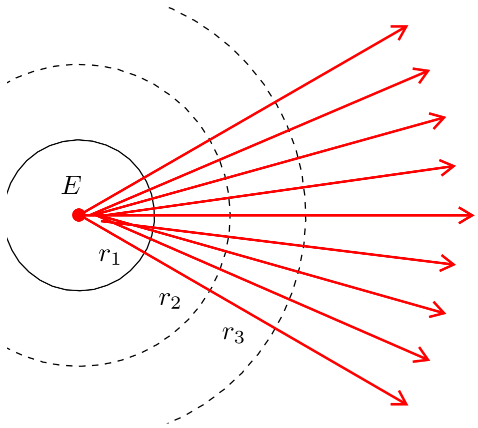

Figure 1.

where $r_3 = 2r_2 = 2r_1$. Furthermore, let us consider three dimensional space, for this is where these laws play their part. Here the red lines represent the energy flux from the source point E. For our sake, let us think of energy, whether it be gravitational, electric, or magnetic, emenating radially outwards. In this system the total energy remains constant. But, as we will see, the energy density, the amount of the energy per unit area, is decreasing at we move outwards from the source point.

Let us suppose that the red lines represent the energy and that the total energy on the surface of a sphere with radius r_1 is exactly the surface area, $4\pi r_1^2$. Let us consider the energy emanating from only one quarter of the surface of sphere. So, for instance, if E were the sun, and I placed a solar panel that fit exactly one quarter of the surface of the sphere at a distance r_1 away, then the solar panel would catch $\frac{4\pi r_1^2}{4} = \pi r_1^2$ of the energy from E.

What if we moved the solar panel to r_2? The key here is that the energy lines are now becoming *less* dense per unit surface area. In other words, the same number of energy lines, i.e. the total energy, is occupying more space. More precisely, the same energy πr_1^2 is now being spread over a quarter of the circumference of the sphere with radius r_2. So the energy is spread over $\frac{4\pi r_2^2}{4} = \pi r_2^2 = 4\pi r_1^2$ rather than πr_1^2 for the sphere of radius r_1. The surface area on the larger sphere is now $\frac{\pi r_1^2}{4\pi r_1^2} = \frac{1}{4}$ as dense with energy. So if we created a new solar panel with surface area $4\pi r_1^2$ then we could capture the same amount of energy as if we placed a solar panel with surface area πr_1^2 on the smaller sphere. However, if we just moved the solar panel to fit the larger sphere, it would only occupy one quarter of the $4\pi r_1^2$ over which the energy has spread out. But each quarter of the $4\pi r_1^2$ over which the energy has spread out contains $\frac{1}{4}$ of the total energy. Thus, the energy catpured by the solar panel has decreased by an order of 4. Similarly, if we moved the solar panel four times further away, then the density of the energy would decrease to 16 times the orginal density of the small sphere and the energy captured by the solar panel would only be $\frac{1}{16}$ of the original.