Conference on Monte Carlo Methods: Theory and Applications

Brown University, April 25-26

Abstracts

Importance sampling algorithms for failure recovery probabilities in computing and data transmission

ASMUSSEN, SØREN *Aarhus University*

A task with length T having distribution F may fail before completion and then has to be restarted (multiple restarts may occur). Asymptotics for the tail $P(X > x)$ in the case of a general failure time distribution G has been derived by Asmussen, Fiorini, Lipsky, Rolski & Sheahan (MOR 2008/09) and by Jelenkovic & Tan. In this talk we study the problem of efficient simulation of this tail by importance sampling when G is exponential. The problem is fairly standard when $T = t$ is deterministic and involves exponential tilting via a Cramér type root. For a random T , we study both a light-tailed and a heavy-tailed scenario. In both, the scheme is the standard one, looking for conditional distributions given $X > x$. In the light-tailed case, the asymptotic distribution of T given $X > x$ is asymptotically a shifted exponentially tilted Gumbel (Fisher-Tippett) distribution. Using this as importance distribution alone is, however, not sufficient to obtain an efficient algorithm, but one has to involve the algorithm using the Cramér root on a set of vanishing probability under the importance distribution. In the heavy-tailed case, the relevant asymptotic distribution of T is a shifted and scaled Pareto. However, this distribution has no mass on an interval of the form $[0, a(x)]$ whereas F does, so the relevant absolutely continuity requirement fails. This is resolved by a supplementary conditioned limited theorem for T given $X > x$ and $T < a(x)$. We also discuss to which extent the multiple root finding in the algorithms can be avoided by inserting an approximation for the Cramér root.

Importance Sampling Methodology for Multidimensional Heavy-tailed Random Walks

BLANCHET, JOSÉ *Columbia University*

We consider the problem of developing efficient rare-event simulation methodology for multidimensional regularly varying random walks. These types of multidimensional objects capture stylized features that often arise in heavy-tailed analysis and that make the development of large deviations approximations challenging. For instance, such features can be appreciated in the estimation of small first-passage time probabilities for which large deviations analysis involves an interesting interplay among the drift of the process, the asymptotic dependence structure of the coordinates for large values of the increments and the geometric position of a target set. We shall propose an approach for constructing importance sampling estimators whose coefficient of variation is bounded as the probability of interest becomes small. The development of the estimators hinges on the construction of a Lyapunov inequality that provides a bound for the second moment of the estimator. We explain how to construct such Lyapunov inequality based on a set of hyperplanes that are obtained depending on the geometry of the set and the drift of the process. We shall also discuss implementation issues related to the path generation of the sampler and apply our construction to an example involving a random walk with t distributed increments. (This is joint work with Jingchen Liu.)

An Efficient Computational Approach for Prior Sensitivity Analysis and Cross-Validation

BORNN, LUKE *Los Alamos National Labs*

Prior sensitivity analysis and cross-validation are important tools in Bayesian statistics. However, due to the computational complexity of existing methods, these techniques are rarely used. In this talk, we show how it is possible to use sequential Monte Carlo to create an efficient and automated algorithm to perform these tasks. We apply the algorithm to the creation of regularization path plots and to check the sensitivity of the tuning parameter in g-Prior model selection. We then demonstrate the algorithm applied to cross-validation and use it to select the shrinkage parameter in Bayesian penalized regression.

On the mean field particle interpretations of Feynman-Kac path integrals

DEL MORAL, PIERRE *Université Bordeaux*

We provide a mean field particle interpretation of Feynman-Kac distribution flows on path spaces in terms of interacting particle systems and genealogical tree based models. We discuss both some theoretical aspects and some application domain areas of this new class of stochastic algorithms, including rare event simulation, nonlinear filtering as well as combinatorial counting and the sampling of complex Boltzmann-Gibbs measures.

Particle Markov Chain Monte Carlo

DOUCET, ARNAUD *University of British Columbia*

Markov chain Monte Carlo (MCMC) and Sequential Monte Carlo (SMC) methods have emerged as the two main tools in computational statistics to sample from high-dimensional probability distributions. Although asymptotic convergence of MCMC algorithms is ensured under weak assumptions, the performance of these latters is unreliable when the proposal distributions used to explore the space are poorly chosen and/or if highly correlated variables are updated independently. We show here how it is possible to build efficient high-dimensional proposal distributions using SMC methods. This allows us to design effective MCMC algorithms in complex scenarios where standard strategies fail. We demonstrate these algorithms on a non-linear non-Gaussian state-space model, a stochastic kinetic model and Dirichlet process mixtures.

Sensitivity Estimates for Levy-Driven Models in Finance

GLASSERMAN, PAUL *Columbia University*

Levy processes with jumps and, more generally, stochastic differential equations driven by Levy processes have been proposed as models for financial data to address shortcomings of diffusion models. These models have been used, in particular, to match market prices of options and other derivative securities. Some options can be valued through closed-form expressions or numerical transform inversion, but others require Monte Carlo simulation. Hedging options requires the calculation of price sensitivities, so we investigate the estimation of sensitivities by Monte Carlo in Levy-driven models. We consider applications of the likelihood ratio method (LRM) and pathwise differentiation. As a benchmark, we analyze the error in LRM estimates when the relevant probability densities are known only through their Laplace transforms and the score function is evaluated through numerical transform inversion. We then analyze approximations, including estimates based on compound Poisson approximations and saddlepoint approximations. This is joint work with Zongjian Liu.

Importance Sampling for Derivatives Pricing with Sample Path Large Deviations

GUASONI, PAULO *Boston University*

Pricing derivative contracts which deliver large payouts on rare events, or no payoff at all, is challenging even with Monte Carlo methods. Importance sampling can substantially improve simulation performance, but its success requires a change of probability which reduces variance, without increasing the cost of each simulation. Among probabilities obtained by a deterministic change of drift, which entail no computational overhead, Sample Path Large Deviations suggest natural candidates for asymptotic optimality, in terms of solutions to classical variational problems. We investigate the assumptions which justify this approach in the Black-Scholes model and in the Heston stochastic volatility model.

Inverse Problems in Dynamical Systems: Data Assimilation

STUART, ANDREW *Warwick University*

Data assimilation is a subject whose genesis has been driven primarily by applied researchers in the atmospheric and geophysical sciences. At its heart is a very high dimensional inverse problem in dynamical systems. In order to reveal the inherent mathematical structure I will formulate this as a Bayesian inference problem on function space, and show that this problem is well-posed. I will then demonstrate how this formulation leads to the construction of efficient MCMC-based sampling methods. The basic idea underlying these methods is that MCMC methods defined on function space are robust under discretization.

Adaptive Monte-Carlo sampling, principal curves and centroidal Voronoi tesselations

VANDEN-EIJNDEN *New York University*

The computation of the mechanism and the rate of rare reactive events in complex dynamical systems requires specific sampling strategies. Example of such events include conformation changes of molecules, nucleation event during phase transition, or chemical reaction. The simulation of these events poses computational challenges because of the wide disparity of time scales involved. In this talk, I will present a numerical technique, the finite-temperature string method (FTS), which permits to address these challenges. FTS computes a discretized approximation of the principal curve associated with the equilibrium distribution of the system (i.e., “the curve which is its own expectation”) by decomposing the system into Voronoi cells whose generating points is also these centroid of these cells. The computation is achieved by adaptive MC sampling in a set of evolving Voronoi cells. I will give the details of how this computation is done, and explain how the result of this computation allows one to compute the rate and pathways of rare events. These results will be illustrated via examples.

Importance Sampling for Jackson Networks

WANG, HUI *Brown University*

A commonly used technique to increase the efficiency of Monte Carlo rare event simulation is importance sampling (IS). Using a recently developed game/subsolution approach, we construct simple and efficient state-dependent IS for simulating buffer overflows in stable open Jackson networks. The sampling distributions do not depend on the particular event of interest, and hence overflow probabilities for different events can be estimated simultaneously. A by-product of the analysis is the identification of the minimizing trajectory for the calculus of variation problem that is associated with the sample-path large deviation rate function.