Conference schedule and abstracts

Thursday, April 20

9.30–10.30 Registration, Room 110, 182 George St.
10.30-11.30 Bob Pego in Room 190, Barus-Holley
11.30-12.30 Dave Levermore
2.30–3.30 Tai-Ping Liu, Room 190, B-H.
3.30–4.30 Reception in Kassar House lobby.
4.30-5.30 Craig Evans, Foxboro Auditorium.

Friday, April 21

8:30-9:00: Coffee, Room 110, 182 George St.
9:00-10:00: Nader Masmoudi, Room 190, B-H.
10:00-11:00: Gui-Qiang Chen
11:30-12:30: Misha Feldman
3:00-4:00: Craig Evans, Foxboro Auditorium.
4:00-5:00: Reception in Kassar House lobby.
5:00-6:00: Craig Evans, Foxboro Auditorium.

Saturday, April 22

8:30-9:00: Coffee, Room 110, 182 George St.
9:00-10:00: Milton Lopes Filho, Room 190 B-H.
10:00-11:00: David Ambrose
11:30-12:30: Tom Hou
2:00-3:00: Vladimir Sverak
3:00-4:00: Gene Wayne
4:30-5:30: Ping Zhang

Sunday, April 23

8:30-9:00: Coffee, Room 110, 182 George St.
9:00-10:00: Zhiwu Lin, Room 190 B-H.
10:00-11:00: Alberto Bressan
11:00-noon: Peter Constantin

Distinguished Lecture Series

Craig Evans *Calculus of variations in the sup norm.*
In this series of three lectures I will discuss how to extend the conventional calculus of variations to a "calculus of variations in the sup-norm", and I will explain why it is interesting to do so.

In particular we will see that the conventional Euler-Lagrange equations transform into fascinating new sorts of PDE that characterize "absolute minimizers" for sup-norm energies. These new equations are nonlinear, second-order and elliptic, but are so degenerate that they behave somewhat like first-order PDE and in particular have "characteristics".

Lec 1. Overview of sup-norm variational problems.

Lec 2. Fine properties of infinitely harmonic functions.

Lec 3. Other energies, Aronsson's equation.

Lefschetz Center Conference Abstracts

David Ambrose *Short-time well-posedness of free surface problems in fluids.* In this talk, I will describe my proofs of short-time well-posedness of various free surface problems in fluids. This will include well-posedness of vortex sheets with surface tension, water waves, and Hele-Shaw flows. I will describe a common framework for treating each of these problems, and will demonstrate how to perform energy estimates for each problem.

Alberto Bressan *Impulsive control of Lagrangian systems and locomotion in fluids.* The talk will present a survey of the theory of impulsive control of finite dimensional Lagrangian systems. In these models, some of the coordinates can be arbitrarily prescribed by a controller as functions of time. It is assumed that these controls are implemented by means of frictionless constraints. This uniquely determines the time evolution of the remaining free coordinates.

After reviewing basic mathematical theory, we shall discuss some infinite dimensional examples, where bodies immersed in fluids achieve locomotion by changing their shape.

Gui-Quiang Chen *Transonic Flow, Shock Reflection, and Free Boundary Problems (I)* In this talk we will start with various shock reflection phenomena and their fundamental scientific issues. Then we will describe how the shock reflection problems can be formulated into free boundary problems for nonlinear partial differential equations of mixed-composite type. Finally we will discuss some recent developments in attacking the shock reflection problems, including our results on shock reflection with Mikhail Feldman.

Peter Constantin *Regularity for some nonlinear Fokker-Planck Navier-Stokes systems.* I will describe coupled systems in which microscopic stresses drive Navier-Stokes equations. The stresses are obtained from solutions of a nonlocal, nonlinear Fokker-Planck equation driven by the fluid. The system has a beautiful structure, with non-trivial energetics and multiple steady solutions. The strategy of proof of regularity yields some bounds for the Navier-Stokes with nearly singular forcing.

Misha Feldman *Transonic Flow, Shock Reflection, and Free Boundary Problems (II)* In this talk we will discuss a recent result and its proof, joint with Gui-Qiang Chen, on the existence of global solutions of shock reflection by large-angle wedges for potential flow. The approach includes techniques to handle free boundary problems, degenerate elliptic equations, and corner singularities.

Milton Lopes Filho *Two dimensional incompressible flow around a small obstacle.* In this talk, we will examine the asymptotic behavior of incompressible flow in the exterior of a small obstacle, both in the viscous and in the ideal cases. Our objective is to describe the results, highlighting the main difficulties in their proofs and the sharp contrast between the two cases.

Tom Hou *The Interplay between Local Geometric Properties and the Global Regularity of the 3D Incompressible Euler Equations.* Whether the 3D incompressible Euler equation can develop a finite time singularity from smooth initial data has been an outstanding open problem. Here we review some existing computational and theoretical work on possible finite blow-up of the 3D Euler equation. We show that there is a sharp relationship between the geometric properties of the vortex filament and the maximum vortex stretching. By exploring this local geometric property of the vorticity field, we have obtained a global existence of the 3D incompressible Euler equations provided that the unit vorticity vector and the velocity field have certain mild regularity property in a very localized region containing the maximum vorticity. Further, we perform large scale computations of the 3D Euler equations to re-examine the alleged finite-time blowup of the two antiparallel vortex tubes, which has been considered as one of the most promising candidates for a finite-time blowup of the 3D Euler equations. Our numerical studies indicate that the maximum vorticity does not grow faster than double exponential in time. The velocity field and the enstrophy remain bounded throughout the computations. The vortex lines near the region of the maximum vorticity are relatively straight. This local geometric regularity of vortex lines seems to be responsible for the dynamic depletion of vortex stretching.

Dave Levermore *Fluid Dynamics from Boltzmann Equations.* We survey some recent results that establish the validity of fluid dynamical systems from the Boltzmann equation. The starting point will be the DiPerna-Lions theory of global solutions to the Boltzmann equation. The target fluid systems will be incompressible and weakly compressible. A new multiple time-scale result will be presented that unifies the acoustic and Stokes limits. Open problems will be discussed.

Zhiwu Lin *Stability of ideal plane flows.* Ideal plane flows are incompressible inviscid two dimensional fluids, described mathematically by the Euler equations. Infinitely many steady states exist. The stability of these steady states is a very classical problem initiated by Rayleigh in 1880. Nevertheless, progress in its understanding has been very slow. I will discuss several concepts of stability and some linear stability and instability criteria. In some cases nonlinear stability and instability can be showed to follow from linear results.

Tai-Ping Liu *Solving Boltzmann equation, the Green's function approach.* With Shih-Hsien Yu, we study the nonlinear waves and boundary behaviors of the solutions to the Boltzmann equation. Our approach is based on the pointwise estimates for the Green's function around a global Maxwellian state. We construct and estimate the Green's function around nonlinear waves and with boundary using the combination of weighted energy method and the Green's functions around global Maxwellian states. Applications to the initial-boundary value problem and to the stability of shock waves will be presented.

Nader Masmoudi *The Nonlinear Schrodinger limit of some Zakharov type systems.* The Klein-Gordon-Zakharov system describes the interaction between Langmuir waves and ion sound waves in plasmas. In this talk we will discuss the high frequency and subsonic limit of the Klein-Gordon-Zakharov system for very general initial data. At the limit we get a Nonlinear Schrodinger system. This work is in collaboration with Kenji Nakanishi.

Bob Pego *Incompressible Navier-Stokes dynamics in bounded domains—a new approach for analysis and computation.* Based upon a new, sharp estimate for the commutator of the Laplacian and Helmholtz projection operators, we show that the pressure gradient is bounded in L2 norm by the viscosity term times a constant less than one, up to lower order terms. By consequence, NSE can be regarded as a perturbed diffusion equation, rather than a perturbed Stokes system. This leads to stability results for discretization schemes that (a) provide simple proofs of existence and uniqueness of local strong solutions, and (b) help explain the success of recently developed numerical methods that are fast, accurate near boundaries, and simple and flexible in structure. This is joint work with Jian-Guo Liu and Jie Liu (Maryland).

Vladimir Sverak *On Landau's solutions of the Navier-Stokes equations* In 1944 L.D.Landau calculated a very interesting family of explicit solutions of the steady-state 3d Navier-Stokes equations. The solutions are derived under certain assumptions of symmetry, which reduce the Navier-Stokes equations to a system of ODEs. We investigate what happens when some of the symmetry conditions are dropped (and we have to deal with PDEs.) Possible implications of these calculations for more general classes of solutions will also be discussed.

Gene Wayne *Stability of Burgers' Vortices in the Three-dimensional Navier-Stokes equations.* I will describe joint work with Th. Gallay of the Univ. of Grenoble in which we prove the existence of non-axisymmetric analogues of the Burgers vortex solution of the Navier-Stokes equation. These solutions exist for arbitrarily large values of the Reynolds number. In addition we show that for small Reynolds number these solutions are stable. More precisely, we prove that any solution of the three-dimensional Navier-Stokes equations whose initial condition is a small perturbation of a Burgers vortex will converge toward another Burgers vortex as time goes to infinity, and we give an explicit formula for computing the change in the circulation number (which characterizes the limiting vortex completely.)

Ping Zhang *On the global wellposedness to the 3-D incompressible anisotropic Navier-Stokes equations.* Corresponding to the wellposedness result for the classical 3-D Navier-Stokes equation with initial data in the scaling invariant Besov space, here we consider a similar problem for the 3-D anisotropic Navier-Stokes equations (ANS) where the vertical viscosity is zero. In particular, our result implies the global wellposedness of (ANS) with high oscillatory initial data. (This is a joint work with J. Y. Chemin)
