Workshop on Structured Dynamical Systems

Lefschetz Center for Dynamical Systems
Division of Applied Mathematics
Brown University

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Explicit Stationary Solutions to a Class of Multiple-Well Dynamics

by Nicholas Alikakos (University of North Texas)

Abstract: We construct heteroclinic connections in closed form for $2\mathbf{U}_{xx} = W_u(\mathbf{U})$, where $x \in R$,$U(x) \in R^2$, and W is a potential with more than one local minimum. then by performing computer simulations we investigate the relevance of these heteroclinic connections to the dynamics of the vector Allen-Cahn equation $U_t = \epsilon^2 \Delta U - W_u(U)$ in two space dimensions and with periodic boundary conditions. (This is joint work with S. Betelu and Xinfu Chen.)

Perturbations of Periodic Orbits of FDE

by Jack Hale (Georgia Institute of Technology)

Abstract: Consider NFDE(f_0), in the space $C = C([-r, 0], R^n)$ with D linear, bounded, atomic at 0 and exponentially stable; that is, the solutions of

$$\frac{d}{dt} D x_t = 0$$

in $C_D = \{\phi \in C : Df = 0\}$ approach zero exponentially as $t \to \infty$. If NFDE(f_0) has a nondegenerate periodic orbit $\Gamma = \{p_s, 0 \leq s < \omega\}$, where $p(s)$ is a periodic solution of period ω, then Γ is a smooth manifold and one can enquire if it is possible to introduce a convenient coordinate system near Γ which will permit one to obtain the same type of results that are known for ODE on the existence of invariant manifolds in $R \times C$ for the nonautonomous perturbation of NFDE(f_0),

$$\frac{d}{dt} D x_t = f_\epsilon(t, x_t)$$

where $f_\epsilon(t, \phi) \to f_0(\phi)$ uniformly as $\epsilon \to 0$. We show that this possible.

Weak Solutions of the Landau-Lifshitz-Gilbert Equation

by Joy Ko (Brown University)

Abstract: The Landau-Lifshitz-Gilbert equation is the basic evolution equation in micromagnetics, a continuum model for ferromagnetism. This problem can be interpreted as a damped Schrödinger map with a Kähler manifold target. In this talk I will survey recent characterizations of weak solutions to these equations. I will also describe numerical strategies to approximate the dynamics of this equation, with application to the study of finite energy finite-time blowup in two dimensions.

Plane Kolmogorov Flows and Takens-Bogdanov Bifurcation without Parameters

by Stefan Liebscher (Brown University)

Abstract: We consider the Kolmogorov problem of viscous incompressible planar fluid flow under external spatially periodic forcing. Looking for time-independent bounded solutions near the critical Reynolds number, we use the Kirchgässner reduction to obtain a spatial dynamical system on a 6-dimensional center manifold. The dynamics is generated by translations in the unbounded spatial direction. Reduction by first integrals yields a 3-dimensional reversible system with a line of equilibria.

This line of equilibria is neither induced by symmetries, nor by first integrals. At isolated points, normal hyperbolicity of the line fails due to a transverse double eigenvalue zero. We investigate such a "Takens-Bogdanov bifurcation without parameters" by blow-up and averaging techniques. In particular we describe the complete set of all small bounded solutions.

In the classical Kolmogorov problem with doubly symmetric external force, bounded solutions consist of periodic profiles, homoclinic pulses and a heteroclinic front-back pair. In the more general case where one of the symmetries is broken, we encounter a complicated set of multipulse solutions.

This is joint work with Andrei Afendikov and Bernold Fiedler.

Dafermos Regularization and an $L2$ Semigroup for the Hyperbolic Conservation Laws

by Xiao-Biao Lin (North Carolina State University)

Abstract: Riemann solutions for the systems of conservation laws $u_t + f(u)_x = 0$ are self-similar solutions of the form $u = u(x/t)$. Using the change of variables $\xi = x/t, \tau - ln(t)$, Riemann solutions become stationary to the system $u_\tau + (Df(u) - \xi I)u_\xi = 0$. I will introduce a $L2$ semigroup for the linear variational systems around the Riemann solutions of the hyperbolic conservation laws. Eigenvalues of the linear system corresponds to zeros of the determinant of a transcendental matrix. If γ is greater than the largest real parts of the eigenvalues, then the semigroup is of $O(exp(\gamma\tau))$ in a weak sense. This work can be applied to the linear stability of Riemann solutions of conservation laws and the stability of nearby solutions of the Dafermos regularizations, i.e. $u_\tau + (Df(u) - \xi I)u_\xi = \epsilon u_{\xi\xi}$.

Denjoy-Wolff Theorems, Fixed Points in Cones and Hilbert's Projective Metric

by Roger Nussbaum (Rutgers University)

Abstract: The classical Denjoy-Wolff theorem considers complex analytic maps $f : B \to B$, where B is the open unit ball in the complex plane. If f has no fixed points in B, the Denjoy-Wolff theorem asserts that there exists a point w in the boundary of B such that for every z in B, $f^k(z)$, the kth iterate of f acting on z, approaches w as k approaches infinity.

A. F. Beardon has observed that the central point in the D-W theorem is that one has a metric space (G, d) and a map $f : G - - > G$ such that (a) f has no fixed points in G, (b) $d(f(x), f(y)) <= d(x, y)$ for all x and y in G and (c) d satisfies a certain geometrical property near the "boundary" of G. In the case of the classical $D - W$ theorem, $G = B$ and d is the Poincare metric on B; but Beardon also gives a finite dimensional argument which shows that the needed geometrical property of d is satisfied when G is a strictly convex, bounded open subset of a finite dimensional Banach space and d is Hilbert's projective metric on G. However, almost all applications in analysis of Hilbert's projective metric involve sets G which are not strictly convex. Furthermore, X may be infinite dimensional. In this generality one must consider $w(z; f)$, the omega limit set of z under f; and $w(z; f)$ need not be a single point even for simple finite dimensional examples. We shall present a theorem which generalizes to infinite dimensions both Beardon's original result and a related finite

dimensional theorem of Noskov and Karlsson. We shall also state a conjecture concerning the location of $w(z;f)$, and we shall state some theorems which support the conjecture.

Asymptotic Symmetry of Positive Solutions of Parabolic Equations: Bounded Domains Reconsidered

by Peter Polacik (University of Minnesota)

Abstract: We consider nonlinear parabolic equations on nonsmooth domains satisfying certain reflectional-symmetry conditions. We prove that global bounded positive solutions are asymptotically symmetric. Compared with earlier theorems of this kind, we do not assume certain crucial assumptions, such as regularity of the nonlinearity in time. Our method relies on new techniques and gives in some sense optimal results.

Attractors for Semilinear Parabolic Equations on the Circle

by Carlos Rocha (Instituto Superior Tecnico, Lisbon, Portugal)

Abstract: We consider scalar semilinear parabolic equations of the form

$$u_t = u_{xx} + f(u, u_x)\ ,\ x \in S^1 \tag{1}$$

defined on the circle $S^1 = I\!R/2\pi\mathbb{Z}$. Under suitable assumptions on the nonlinearity $f = f(u, u_x)$, such equations generate global dissipative semiflows $T_f(t) : X \to X$ on appropriate function spaces X with a nonempty global attractor $\mathcal{A}_f$. Due to the periodic boundary conditions the semiflow T_f, in general, exhibits nonequilibrium periodic orbits. From a dynamical point of view this provides for a very rich geometric structure of the global attractor $\mathcal{A}_f$.

A typical question that arises in the classification of Morse-Smale attractors is the following: up to a homeomorphism how many different Morse-Smale attractors with N equilibria and P periodic orbits are there?

We consider a permutation characterization for (1) and describe a number of examples of attractors. In particular, we answer the previous classification question enumerating all the attractors with N equilibria and P periodic orbits for $N + 2P \leq 9$.

Stability of Self-Similar Solutions of the Dafermos Regularization of a System of Conservation Laws

by Stephen Schecter (North Carolina State University)

For a system of viscous conservation laws in one space dimension,

$$u_T + f(u)_X = (B(u)u_X)_X, \quad -\infty < X < \infty, \tag{2}$$

the *Dafermos regularization* is the system

$$u_T + f(u)_X = \epsilon T(B(u)u_X)_X. \tag{3}$$

It is usually regarded as an artificial, nonphysical equation because of the factor T in the viscous term. In the variables $x = \frac{X}{T}$, $t = \ln T$, (3) becomes

$$u_t + (Df(u) - xI)u_x = \epsilon(B(u)u_x)_x. \tag{4}$$

In these variables, the Dafermos regularization is actually a natural simplification of (2) written in the same variables, which are natural variables for the study of the asymptotic behavior of solutions of (2).

It is known in many cases that near a Riemann solution $\hat{u}(x)$, $x = \frac{X}{T}$, of the system of conservation laws $u_T + f(u)_X = 0$, with shock waves that satisfy the viscous profile criterion for $B(u)$, there are, for small $\epsilon > 0$, steady-state solutions $u_\epsilon(x)$ of (4). We refer to these smooth solutions as Riemann-Dafermos solutions. Their time-asymptotic stability as solutions of (4) can be studied by linearization. The information thus obtained should be useful in understanding the asymptotic behavior of solutions of (2).

For Riemann-Dafermos solutions near Riemann solutions consisting of n Lax shock waves, with the viscosity $B(u) \equiv I$, it turns out that in an appropriate weighted function space, the essential spectrum of the linearized system lies in $\mathrm{Re}\,\lambda \leq -\delta < 0$, so spectral stability is determined by the eigenvalues. There are fast eigenvalues of order $\frac{1}{\epsilon}$ and slow eigenvalues of order one. The fast eigenvalues correspond to eigenvalues (other than 0) of the viscous profiles for the individual shock waves in the Riemann solution; these have been studied by Zumbrun and collaborators using Evans function methods. The slow eigenvalues are related to inviscid stability conditions that have been obtained by Shochet, Bressan, Lewicka and others for the underlying Riemann solution.

Most of what I will discuss is joint work with Xiao-Biao Lin.

Consider a system of viscous conservation laws in one space dimension,

$$u_T + f(u)_X = (B(u)u_X)_X, \quad -\infty < X < \infty, \tag{5}$$

with constant boundary conditions

$$u(-\infty,T)=u^\ell, \quad u(+\infty,T)=u^r,$$

and some initial condition $u(X,0)=u^0(X)$. It is believed that as $T\to\infty$, solutions typically approach Riemann solutions for the system of conservation laws

$$u_T+f(u)_X=0. \tag{6}$$

The shock waves in the Riemann solution should satisfy the viscous profile criterion for the viscosity $B(u)$.

In the variables $x=\frac{X}{T}$, $t=\ln T$, (5) becomes

$$u_t+(Df(u)-xI)u_x=e^{-t}(B(u)u_x)_x. \tag{7}$$

(The substitution $x=\frac{X}{T}$ is motivated by the fact that Riemann solutions are functions of $\frac{X}{T}$; the substitution $t=\ln T$ converts algebraic decay in T lnto exponential decay in t.)

If we “freeze time” in (7) by setting $\epsilon=e^{-t}$, we obtain

$$u_t+(Df(u)-xI)u_x=\epsilon(B(u)u_x)_x. \tag{8}$$

Returning to (X,T) variables, (8) becomes

$$u_T+f(u)_X=\epsilon T(B(u)u_X)_X. \tag{9}$$

Equation (9) is the Dafermos regularization of the system of conservation laws (6) associated with the viscosity $B(u)$. It is usually regarded as an artificial, nonphysical equation because of the factor T in the viscous term.

It is known in many cases that near a Riemann solution $\hat{u}(x)$, $x=\frac{X}{T}$, of (6), with shock waves that satisfy the viscous profile criterion for $B(u)$, there are, for small $\epsilon>0$, steady-state solutions $u_\epsilon(x)$ of (8) that satisfy the boundary conditions

$$u(-\infty)=u^\ell, \quad u(+\infty)=u^r. \tag{10}$$

We refer to these smooth solutions as Riemann-Dafermos solutions. Their time-asymptotic stability as solutions of (8) can be studied by linearization. The information thus obtained should be useful in understanding Riemann solutions as time-asymptotic states of (5).

For Riemann-Dafermos solutions near Riemann solutions consisting of n Lax shock waves, with the viscosity $B(u)\equiv I$, Xiao-Biao Lin and I have obtained the following results. In an appropriate weighted function space, the essential spectrum of the linearized system lies in Re $\lambda\le-\delta<0$, so spectral stability is determined by the eigenvalues. There are fast eigenvalues of order $\frac{1}{\epsilon}$ and slow eigenvalues of order one. The fast eigenvalues correspond to eigenvalues (other than 0) of the viscous profiles for the individual shock waves in the Riemann solution; these have been studied by Zumbrun and collaborators using Evans function methods. The slow eigenvalues are related to inviscid stability conditions that have been obtained by Shochet, Bressan, Lewicka and others for the underlying Riemann solution.

On the Longtime Dynamics of a Simple Model of the Ocean

by George Sell (University of Minnesota)

To Be Announced

by Sjoerd Verduyn Lunel (University of Leiden, Netherlands)

The Generalized Korteweg-de Vries Equation: Stability of Solitary Waves in the Singular Speed Limit

by Martin Wechselberger (Ohio State University)

Abstract: Pego and Weinstein studied the generalized Korteweg-de Vries equation (gKdV), calculated the Evans function and gave conditions for spectral stability. But this theory cannot be applied in the singular limit c=0 of the wave speed because the Evans function is simply not defined. The main reason is that the absolute spectrum touches the essential spectrum and the profile loses exponential dichotomy properties. We are able to analyse the stability properties of the standing wave by applying the blow-up technique which allows us to extend the Evans function into this singular limit.

Existence and Stability of Traveling Wavefronts in a Hyperbolic Equation with Delay

by Jianhong Wu (York University, Toronto, Canada)

Abstract: We discuss briefly the biological motivation for a hyperbolic equation with delay in modeling time lags in spatial movement of biological species with age structure. We show how a hybrid monotone iteration (solving a coupled system of a second order functional differential equation of mixed type and an integral equation) can be used to obtain the existence of traveling wavefronts, and we establish the stability of wavefronts in weighted spaces using a few Liapunov functionals and some detailed information about the rates of convergence of wavefronts at infinity. This is based on joint work with Genevieve Raugel.

Quasi-periodic Solutions of Nonlinear Schrödinger Equations

by Yingfei Yi (Georgia Institute of Technology)

Abstract: This talk concerns the existence of quasi-periodic solutions in nonlinear Schrödinger equations with various boundary conditions. Both CWB and KAM methods will be discussed and some recent results will be presented.

Perturbations to Homoclinic Orbits for Conservative Systems

by Chongchun Zeng (University of Virginia)

Abstract: Given a dynamical system with a conserved quantity and a homoclinic orbit to a saddle-center fixed point, we consider the defomration of the homoclinic orbit under small conservative perturbations. Under certain nondegeneracy conditions and a rather general assumption that the Hessian of the conserved quantity at the fixed point is positive definite in the center directions, we prove that orbits homoclinic to center manifolds persist. Applications to nonlinear wave equations will be discussed.