1 Introduction

Drug abuse and addiction are prevalent social problems in the United States. In 2017, 19.7 million American adults (aged 12 and older) battled a substance use disorder [1]. Among other addictive substances, Vicodin is the most widely prescribed drug in the United States. Vicodin is a Schedule III narcotic. Research has shown that most abusers who obtain the drug via prescription are unaware of its potential for its physical and chemical dependence [3]. A limited number of medical professionals have received trainings on the risk of Vicodin dependency, which are proven to be effective in preventing Vicodin abuses.

The project examines three mechanistic mathematical models built in [2] for a population introduced to Vicodin by prescription and their dynamics and transition through the stages of medical use, drug abuse, and treatment. Using the three models, the project analyzes whether focusing on abuse prevention methods or treatment methods would be more effective in reducing the number of Vicodin abusers in this population.

All three models are prototypes of the usual SIR model. The SIR model is a simulation of the number of people infected with a contagious illness in a closed population over time. It was proposed Kermack and McKendrick in 1927 to explain the rapid rise and fall in the number of infected patients observed in epidemics. The SIR model is characterized by the compartmental model where $S$ is the number of susceptible people, $I$ is the number of people infected, $R$ is the number of people who have recovered and developed immunity to the infection.

![SIR compartmental model](Figure 1: A typical SIR compartmental model)

Today, the SIR model is still widely used to study infectious diseases and inform control measures of those diseases. For example, [4] uses SIR model to optimize the part of the population to immune.
incorporates sensitivity analysis to measure the degree to which each of the parameters affect the outcome of the model. Sensitivity analysis determines how sensitive the model is to each input parameter and is useful when modeling from experimental data. It effectively determines which parameter should be the focus of attention.

2 Background

The three models consider relapse into the abuse compartment in two ways: two non-linear models incorporating social interaction and one linear without it. The first model is the linear Compartmental Vicodin Transition (CVT) Model without considering social interaction:

\[
\begin{align*}
\frac{dS}{dt} & = \Lambda - (\alpha_1 + \alpha_2)M \\
\frac{dC_1}{dt} & = \alpha_1 M - (\delta + \beta)C_1 \\
\frac{dC_2}{dt} & = \delta C_1 - (\delta + \beta)C_2 \\
\frac{dA}{dt} & = \delta C_2 + (\gamma_1 T - \epsilon A) \\
\frac{dT}{dt} & = \epsilon A - (\gamma_1 + \gamma_2)T
\end{align*}
\]

**Figure 2:** The first model: the linear Compartmental Vicodin Transition (CVT) Model in [2]

In this model, a population of individuals prescribed Vicodin are classified according to level of Vicodin use. The first compartment consists of acute medical users (M), who enter immediately when prescribed the narcotic. Patients leave the population after taking Vicodin for up to three months, or they enter a chronic compartment (\(C_1\)). Further, they move to the \(C_2\) compartment if continuing to take Vicodin for medical reasons, or move to the abuse compartment (\(A\)) if they begin taking the drug in a manner that is inconsistent with the prescribed dosage. Otherwise, if the patients stop taking Vicodin, they leave \(C_2\) and exit the population. Individuals in the abusive compartment can either remain there or enter the treatment compartment (\(T\)), from where they can either leave the population through successful treatment or re-enter the A compartment through relapse.

In the two non-linear models, the paper considers the rate of entrance into the population as a constant through the Social Interaction with Constant Prescription Rate (SIC)
Model and also varying according to the population of Vicodin abusers through the Social Interaction with Abuse-Dependent Prescription Rate (SIAD) Model.

3 Proposed Methodology

First, we will compare and construct the three models provided in [2] and perform numerical simulations with Runge-Kutta 4 Method using Matlab.

Next, we will learn about sensitivity analysis in general context. We will explore both the local methods and the global methods. Within the local methods category, we will focus on forward sensitivity equations, relative sensitivity function and adjoint method.

Finally, we will look at how sensitivity analysis is performed in this research. We will study how the research paper uses two of the local methods: forward sensitivity equations and adjoint method, and possibly perform the other types of sensitivity research: relative sensitivity function in the context of the paper.

By studying the three models and performing sensitivity analysis, we seek to understand whether the models are more sensitive to parameters associated with prevention than parameters associated with treatment. From there, we are able to determine which methods are more effective in reducing the number of Vicodin abusers in the population.

References


