

Concentration of Measure Phenomenon (geometric law of large numbers)

In the statistical setting it states the following: Let f be a Lipschitz function with Lipschitz constant L on the d -sphere. Let P be a normalized Lebesgue measure on the sphere and let X be a random variable uniformly distributed with respect to P . Then,

$$P\{|f(X) - Ef(X)| > t\} \leq c_1 \exp(-c_2 t^2 / L^2)$$

with constants c_1, c_2 independent of f and d . In its simplest form, the phenomenon of concentration of measure just says that every Lipschitz function on a sufficiently high dimensional domain Ω is well approximated by a constant function (Hegland and Pestov 1999, Baxter and Iserles 2003). Thus, there is some chance to treat high dimensional problems despite the curse of dimensionality.

(Griebel, 2005)