The Discrete Conceit: Agent-Based Aggregation Models

Chad Topaz, Macalester College
Andrew Bernoff, Harvey Mudd College

The big picture

- Biological aggregations are social groups displaying collective behavior.
- They matter for pattern formation, biology, algorithms…
- Consider a minimal modeling approach.
- There are hundreds or thousands of published models, and some have been highly influential.
- Choose the right model for the job.

Biological Aggregations

Biological aggregations move in a coordinated manner.

Parrish & Keshet, Nature, 1999

Aggregations can propagate without a leader.

http://youtu.be/iRNqhi2ka9k

Social interactions are key to the formation of groups.

Google Image "locust swarm"

Social interactions are key to the formation of groups.

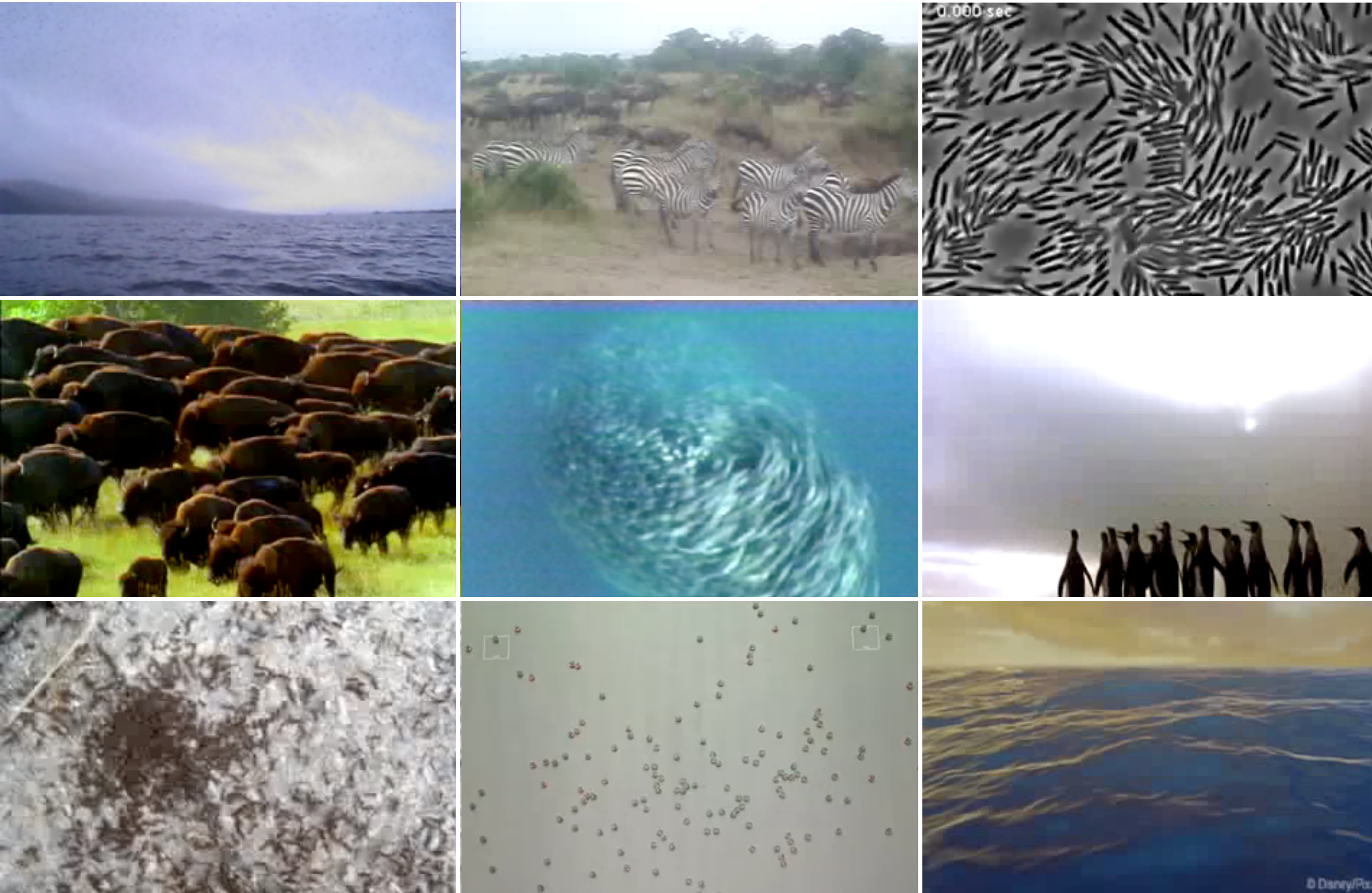
Google Image "locust swarm"

Who cares?

"Social behaviors [that] on short time and space scales lead to the formation and maintenance of groups… lead at larger time and space scales to differences in spatial distributions of populations and rates of encounter and interaction with populations of predators, prey, competitors and pathogens… At the largest time and space scales, aggregation has profound consequences for ecosystem dynamics and for evolution of behavioral, morphological, and life history traits."

- Okubo, Keshet, Grunbaum, "The dynamics of animal grouping" in Diffusion and Ecological Problems, Springer (2001)

Collective motion occurs across the natural and engineered worlds.



Aggregation Models

There are numerous classes of aggregation models

Continuum

Discrete

Kinematic

Dynamic

Deterministic

Stochastic

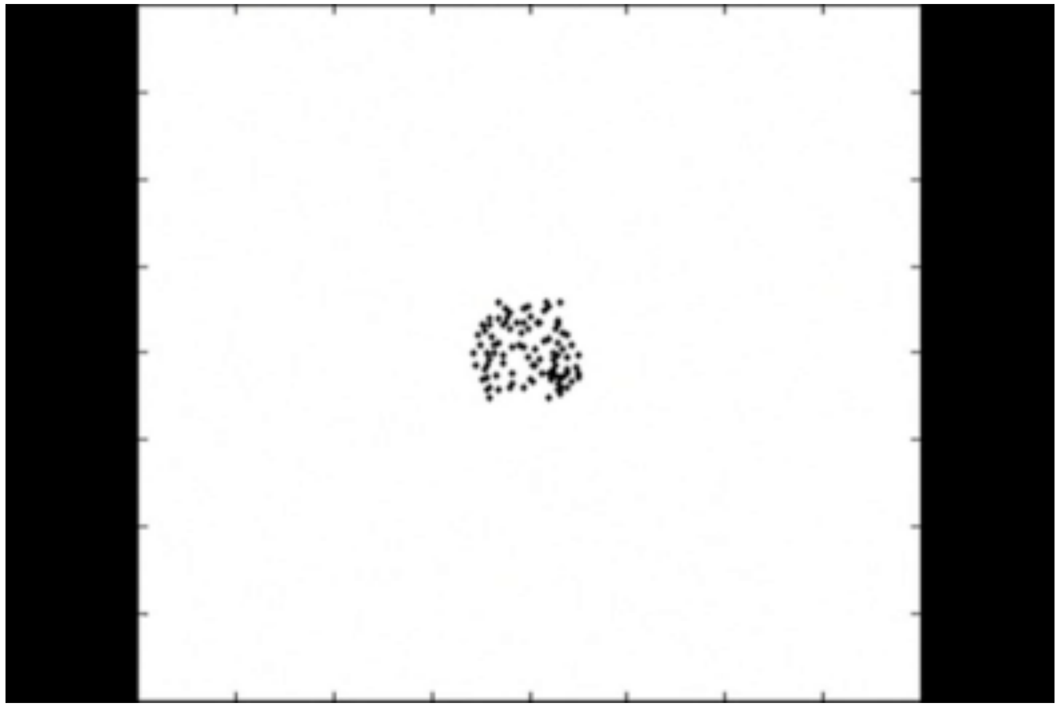
Isotropic

Nonisotropic

Consider the value of parsimony in your models.

Fish neurobiology
Fish behavior
Ocean current profiles
Fluid dynamics
Resource distribution

Self-propulsion
Attraction/repulsion
Alignment



Example 1: Attraction/repulsion

[PDF] Equations descriptive of **fish** schools and other animal aggregations
CM **Breder** - Ecology, **1954** - JSTOR
In an effort to understand better the basic nature of the influences at work in a school of fishes as well as in other less compact aggregations, **Breder** (1951) discussed, in passing, the possibility of applying physical equations to such groups, without going into the matter **...**
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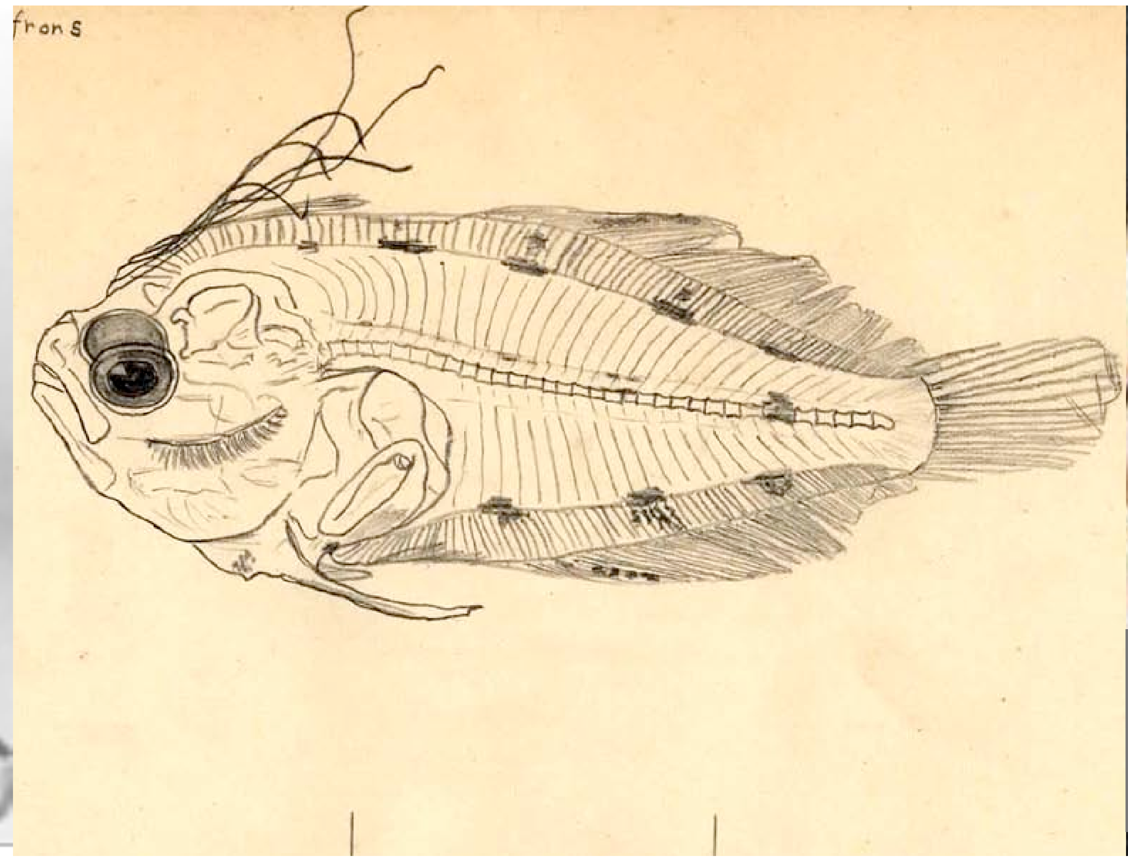
A historical note on Charles M. Breder



His earliest publication, written at 18, concerned photography of local birds and by age 21 he had published 15 popular articles and notes and had started his theoretical and experimental studies on fish locomotion. In 1925 the New York Academy of Sciences awarded him the A. Cressy Morrison Prize for his pioneering and penetrating analysis of fish locomotion…

Example 1: Attraction/repulsion

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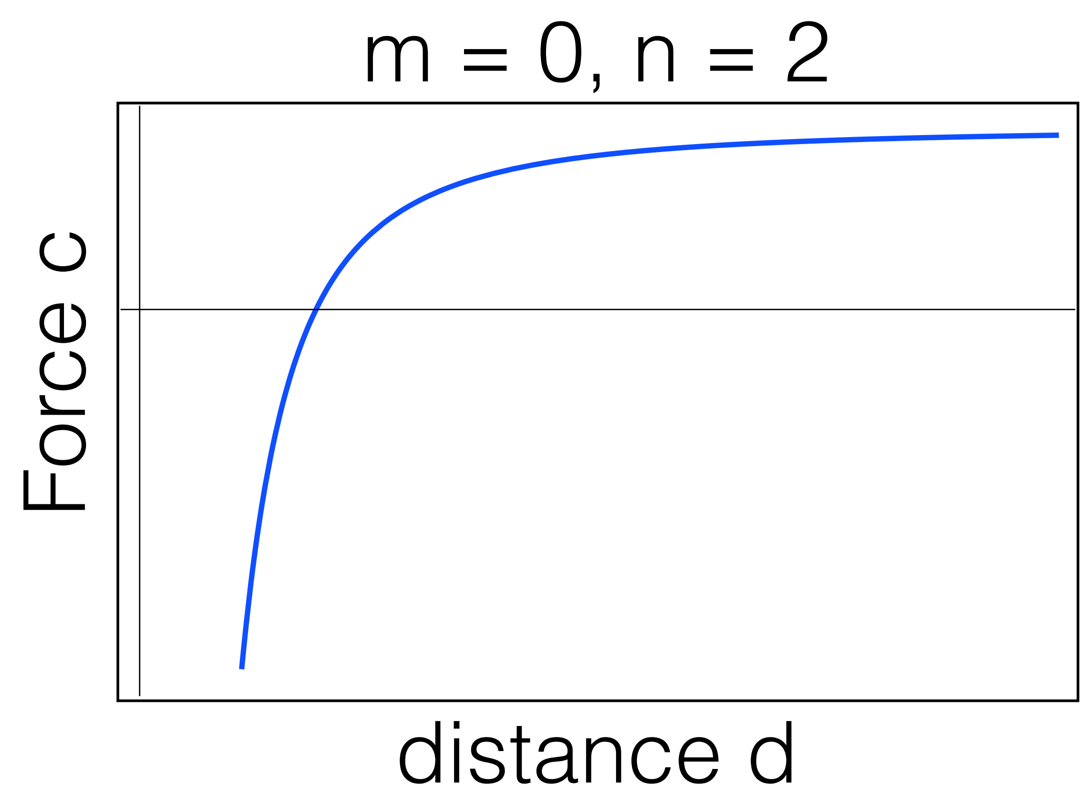
attraction repulsion

$$c = \frac{a}{d^m} - \frac{r}{d^n}$$

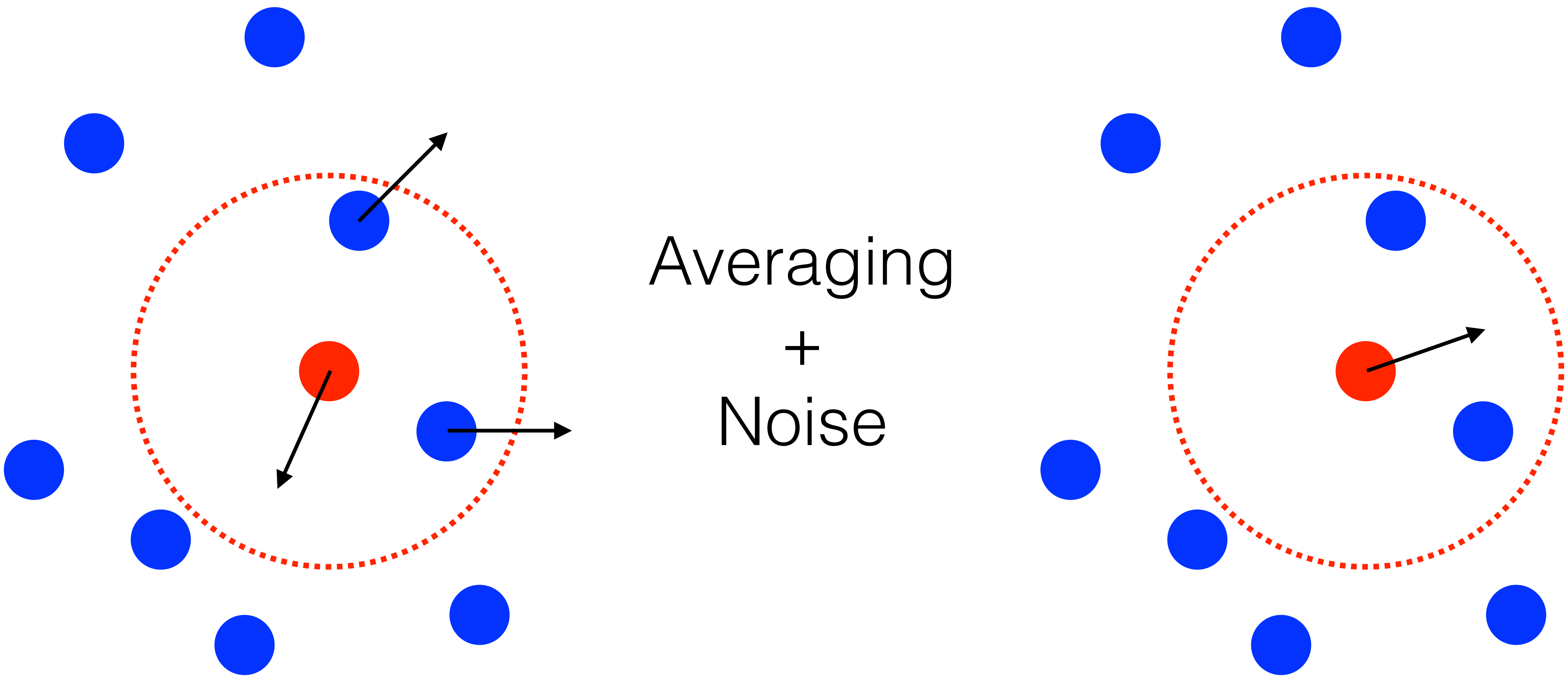
force between organisms/groups distance



Example 2:
Kinematic Alignment

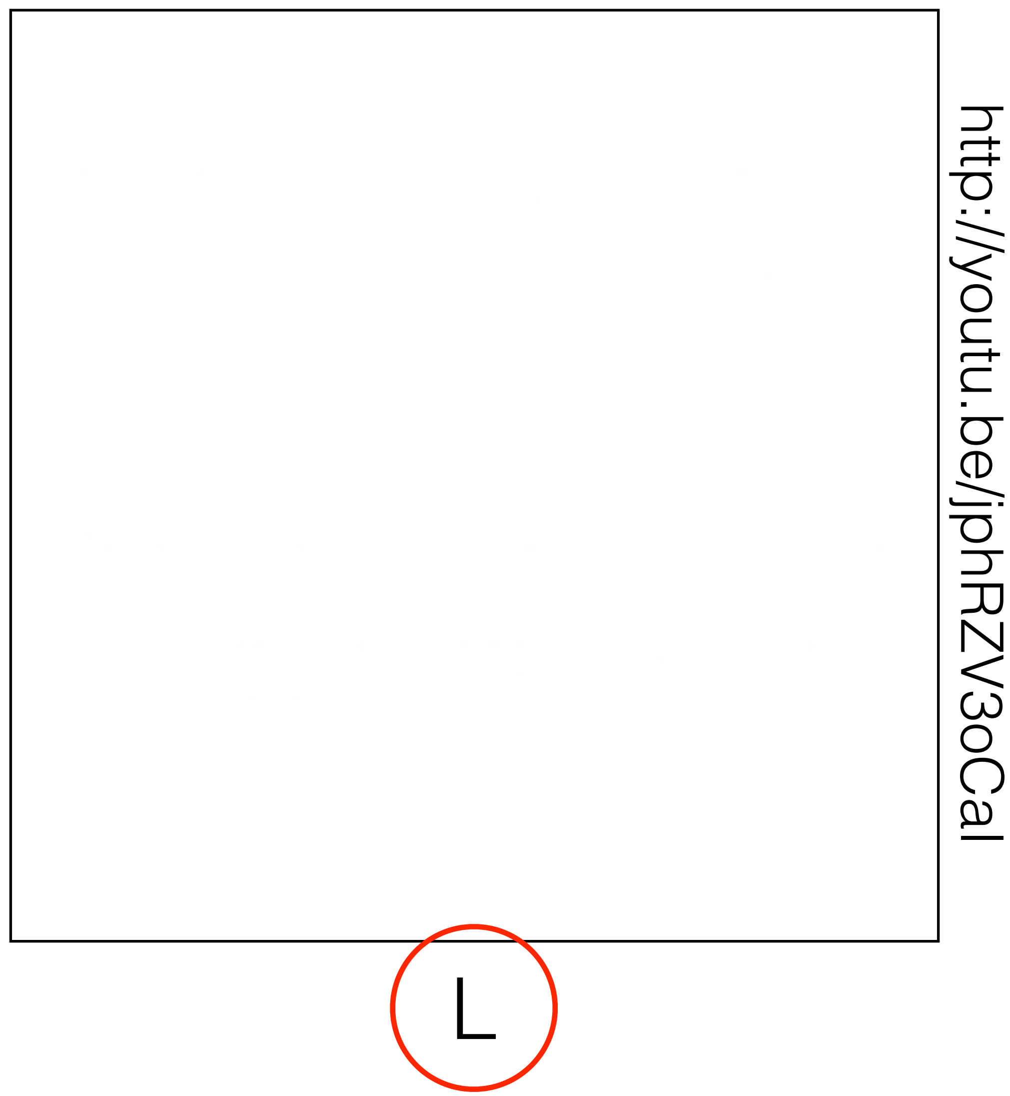
Novel type of phase transition in a system of self-driven particles
T **Vicsek**, A Czirók, E Ben-Jacob, I Cohen, O Shochet - Physical review letters, 1995 - APS
Abstract A simple model with a novel type of dynamics is introduced in order to investigate the emergence of self-ordered motion in systems of particles with biologically motivated interaction. In our model particles are driven with a constant absolute velocity and at each ...
Cited by 3214 Related articles All 23 versions Cite Save

Example 2:
Kinematic Alignment

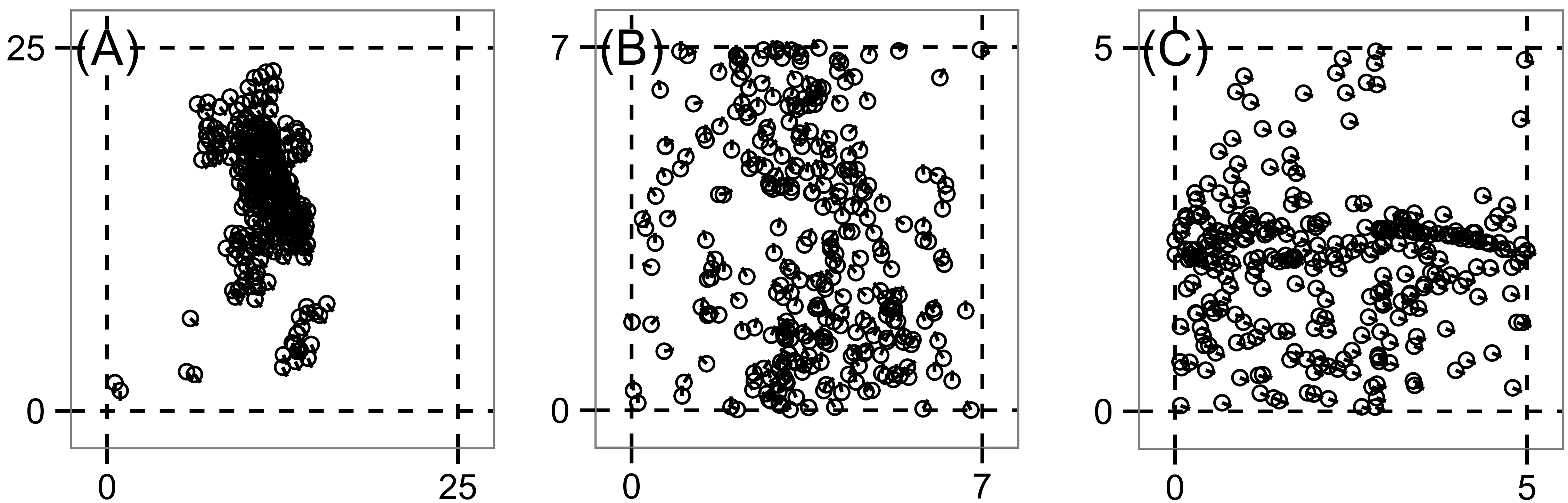


Example 2: Kinematic Alignment

$$
\begin{aligned}
\theta_i &\leftarrow \frac{1}{N} \sum_{|\mathbf{x}_i - \mathbf{x}_j| \le R} \theta_j \\
&\quad + U(-\eta/2, \eta/2) \\
\mathbf{v}_i &\leftarrow v_0 (\cos\theta_i, \sin\theta_i) \\
\mathbf{x}_i &\leftarrow \mathbf{x}_i + \mathbf{v}_i \, \Delta t
\end{aligned}
$$



Example 2: Kinematic Alignment



Clusters

Loose alignment

Strong alignment

Example 3: Dynamic, Self-Propelled Swarmers

Self-propelled particles with soft-core interactions: patterns, stability, and collapse

MR **D'Orsogna**, YL Chuang, AL Bertozzi, LS Chayes - Physical review letters, 2006 - APS

Abstract Understanding collective properties of driven particle systems is significant for naturally occurring aggregates and because the knowledge gained can be used as building blocks for the design of artificial ones. We model self-propelling biological or artificial **...**

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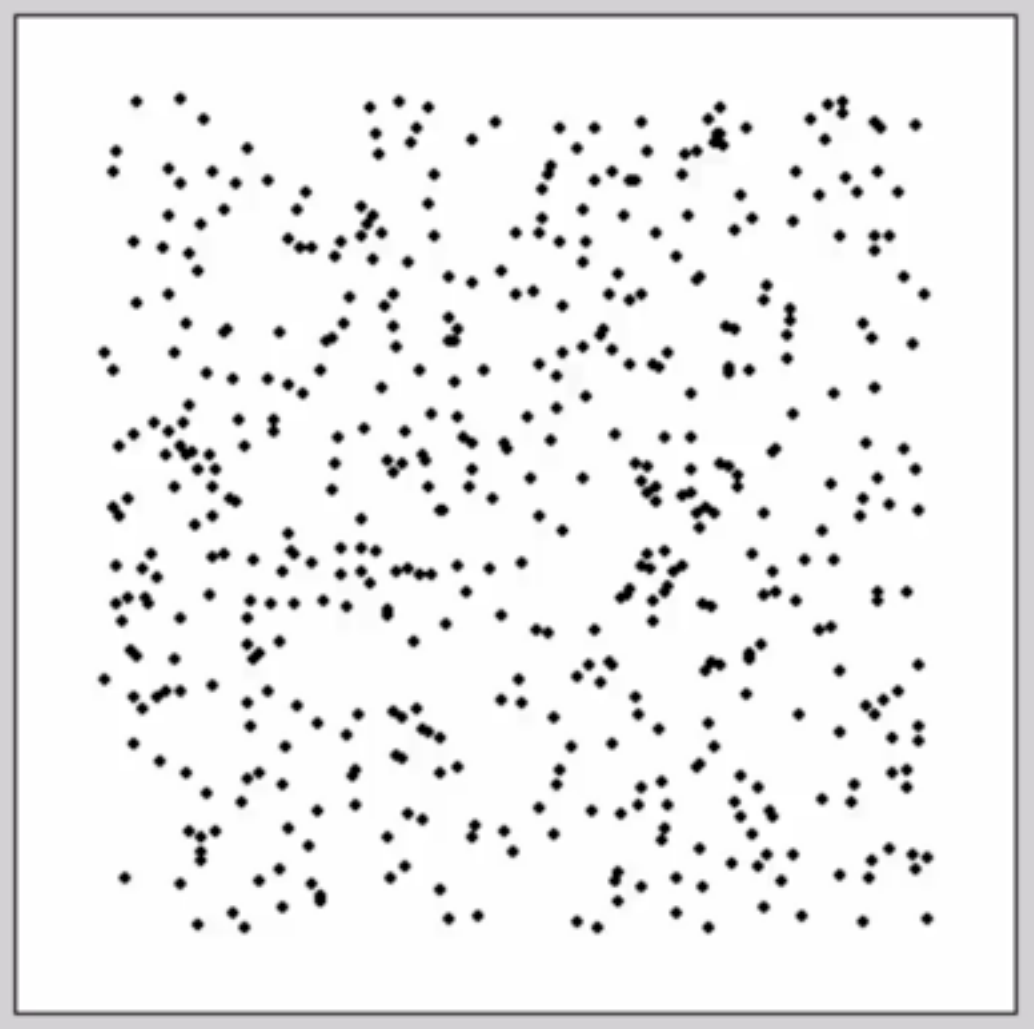
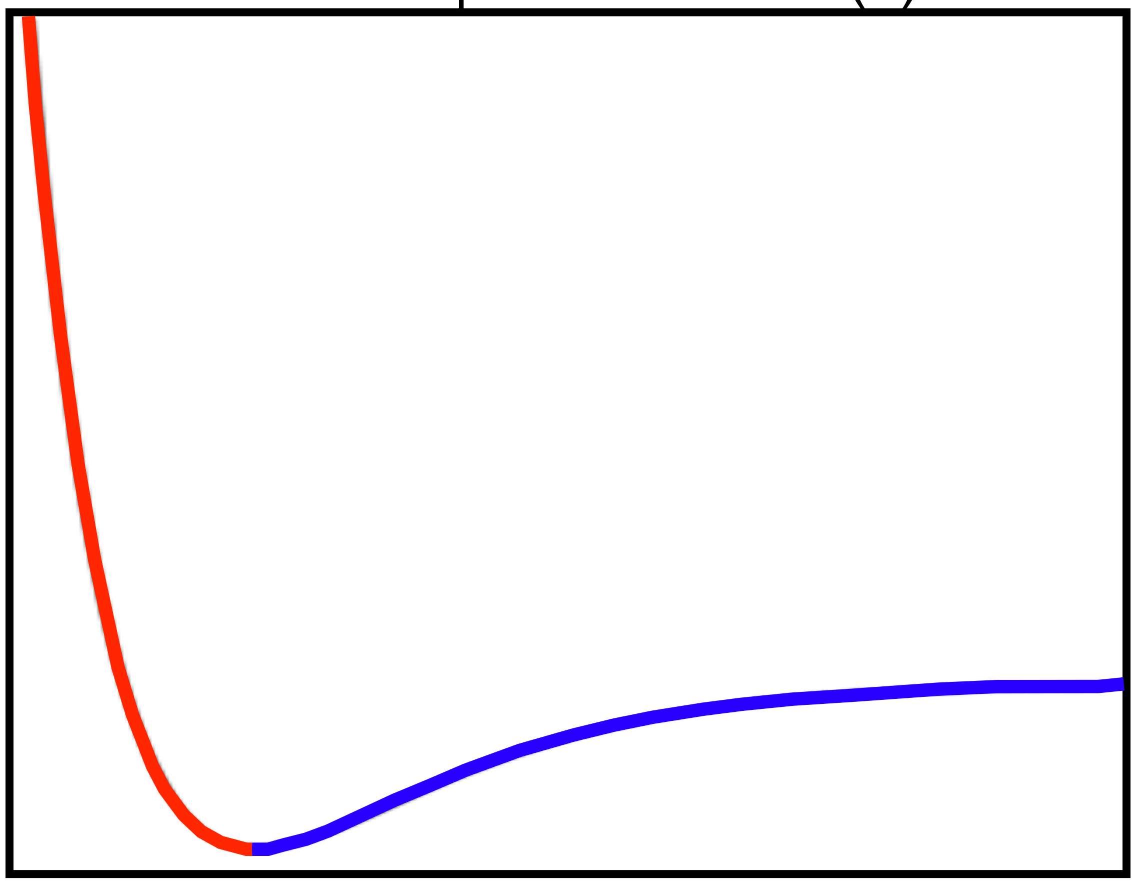
Example 3: Dynamic, Self-Propelled Swarmers

$$
\begin{aligned}
\dot{\mathbf{x}}_i &= \mathbf{v}_i \\
m\dot{\mathbf{v}}_i &= \left(\alpha - \beta|\mathbf{v}_i|^2\right)\mathbf{v}_i - \nabla_i Q_i \\
Q_i &= \sum_{j \neq i} C_r e^{-|\mathbf{x}_i - \mathbf{x}_j|/L_r} \\
&\quad - C_a e^{-|\mathbf{x}_i - \mathbf{x}_j|/L_a}.
\end{aligned}
$$

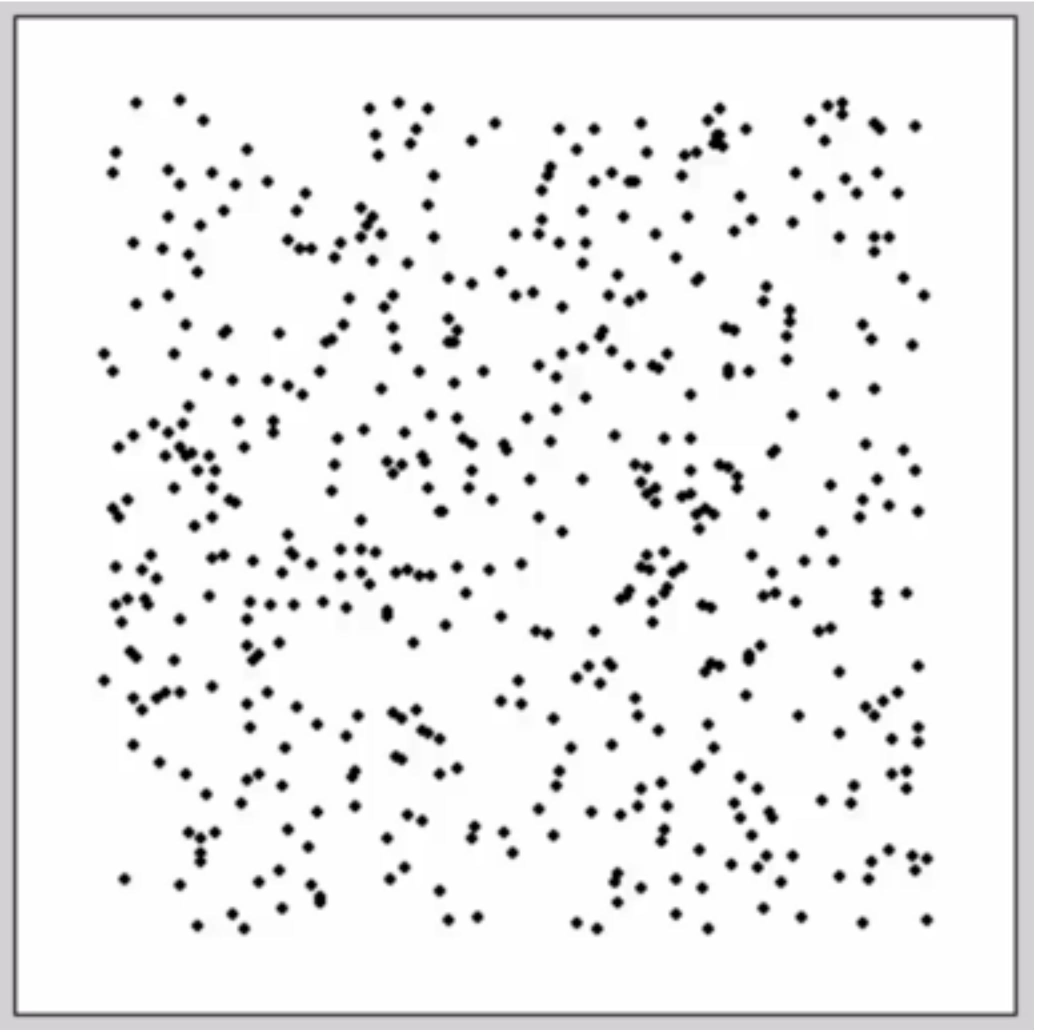


Example 3: Dynamic, Self-Propelled Swarmers

Social potential Q(r)



r = interorganism distance



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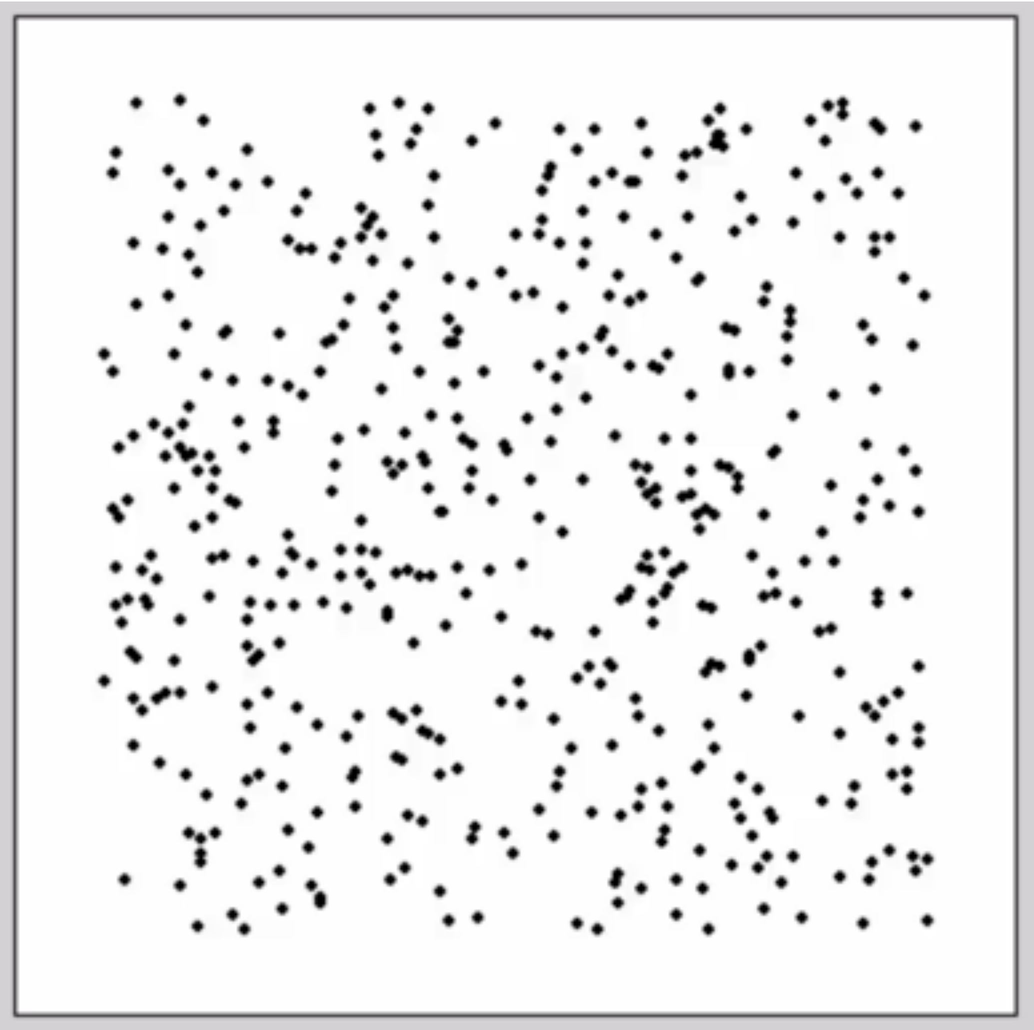
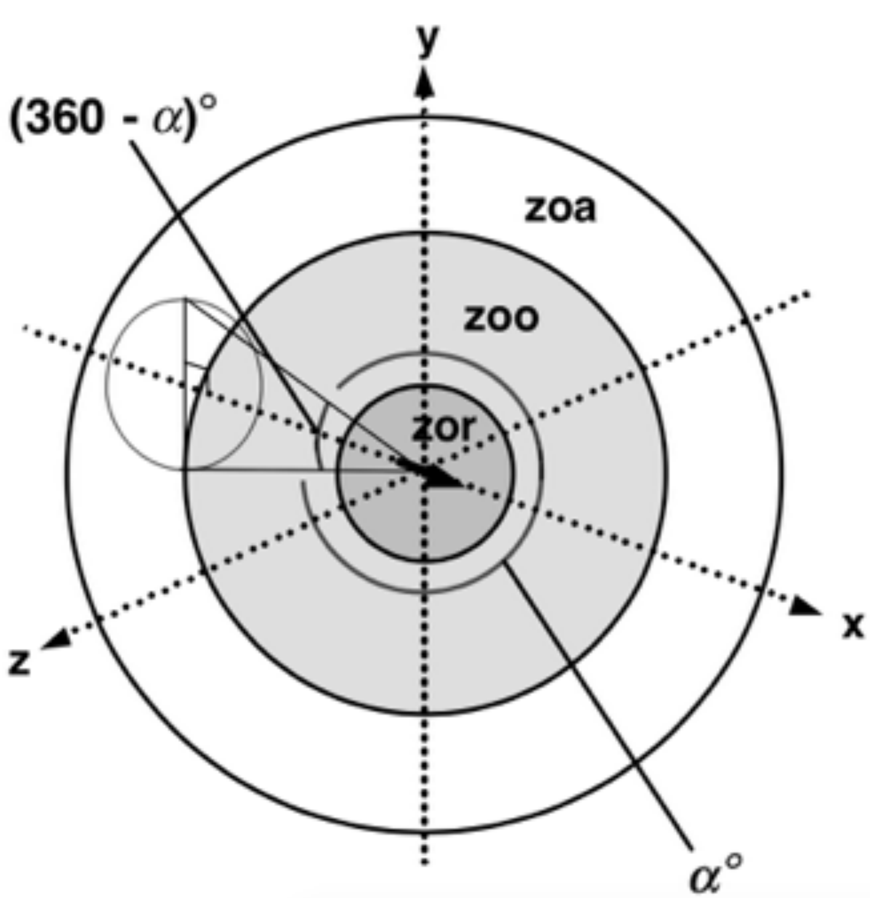


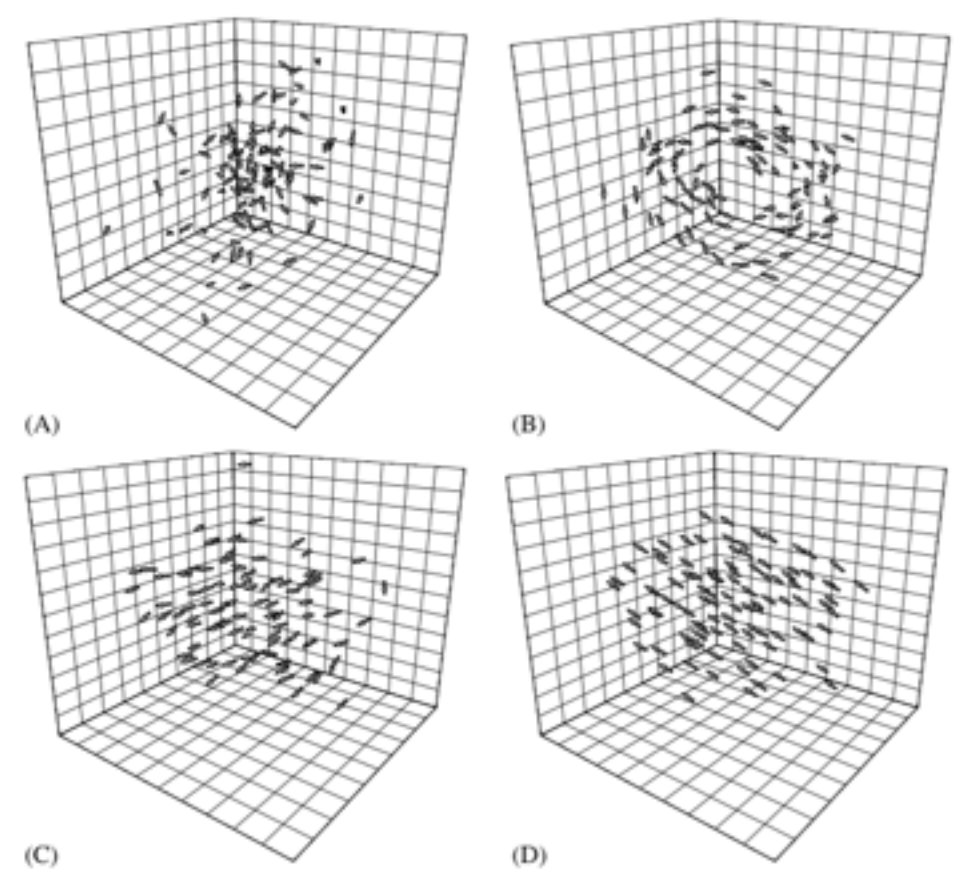
Example 4:
Kitchen Sink

Collective memory and spatial sorting in animal groups

ID **Couzin**, J **Krause**, R James, GD Ruxton… - Journal of theoretical …, 2002 - Elsevier

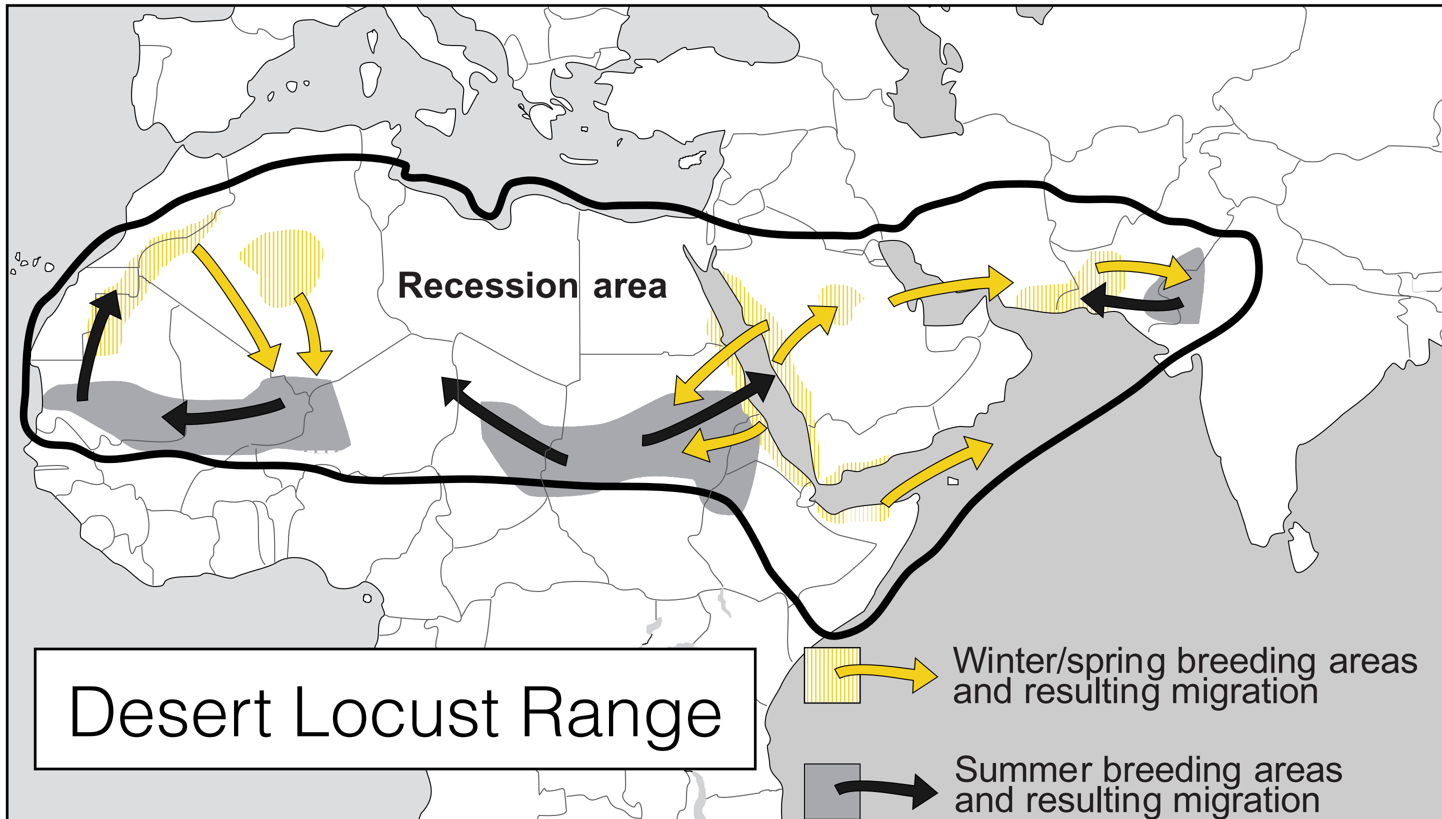
We present a self-organizing model of group formation in three-dimensional space, and use it to investigate the spatial dynamics of animal groups such as fish schools and bird flocks. We reveal the existence of major group-level behavioural transitions related to minor **...**



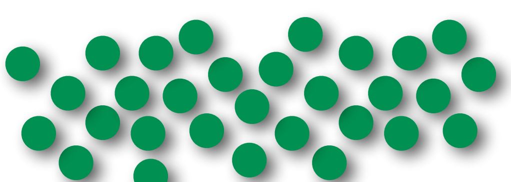


Case Study 1:
Locusts

Locusts = devastating



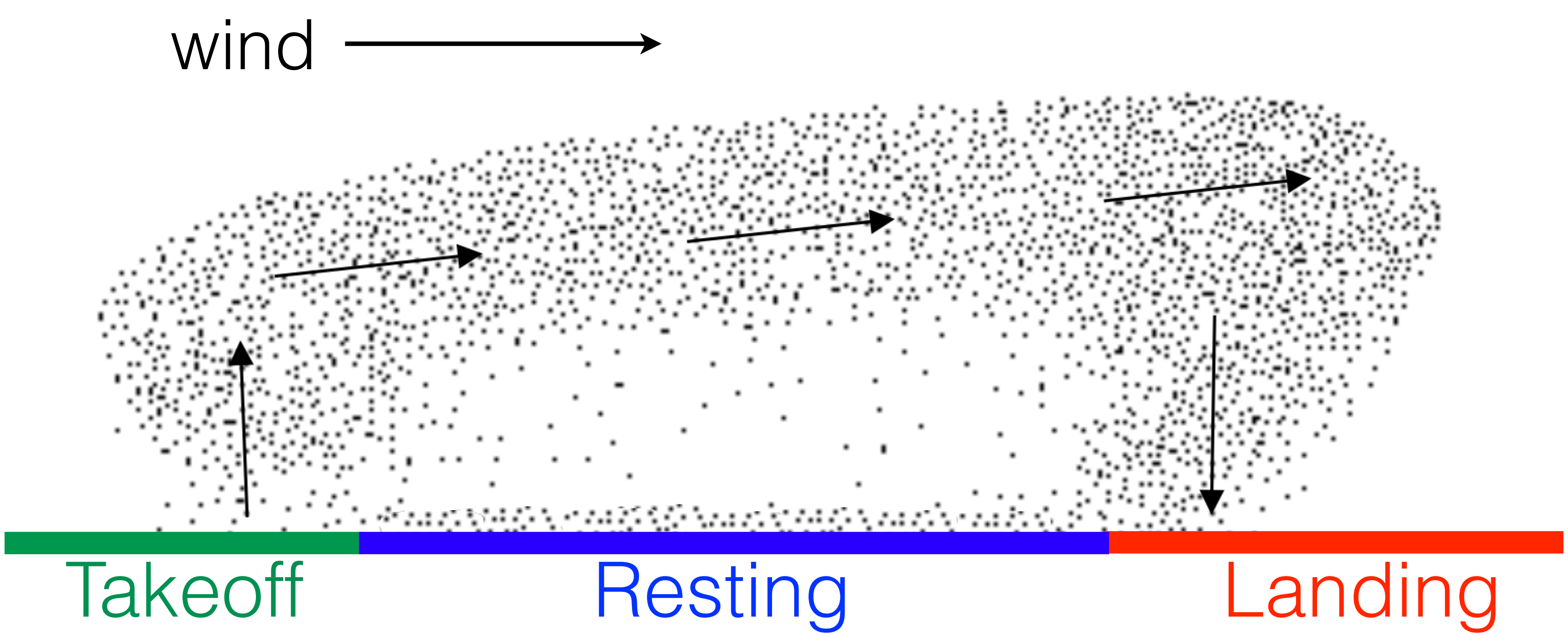
10^{10} locusts



100 km^2

10 - 100 km/day

Locust swarms migrate with a rolling motion



Uvarov, Grasshoppers and Locusts (1977)

Model it.

Eur. Phys. J. Special Topics **157**, 93–109 (2008)

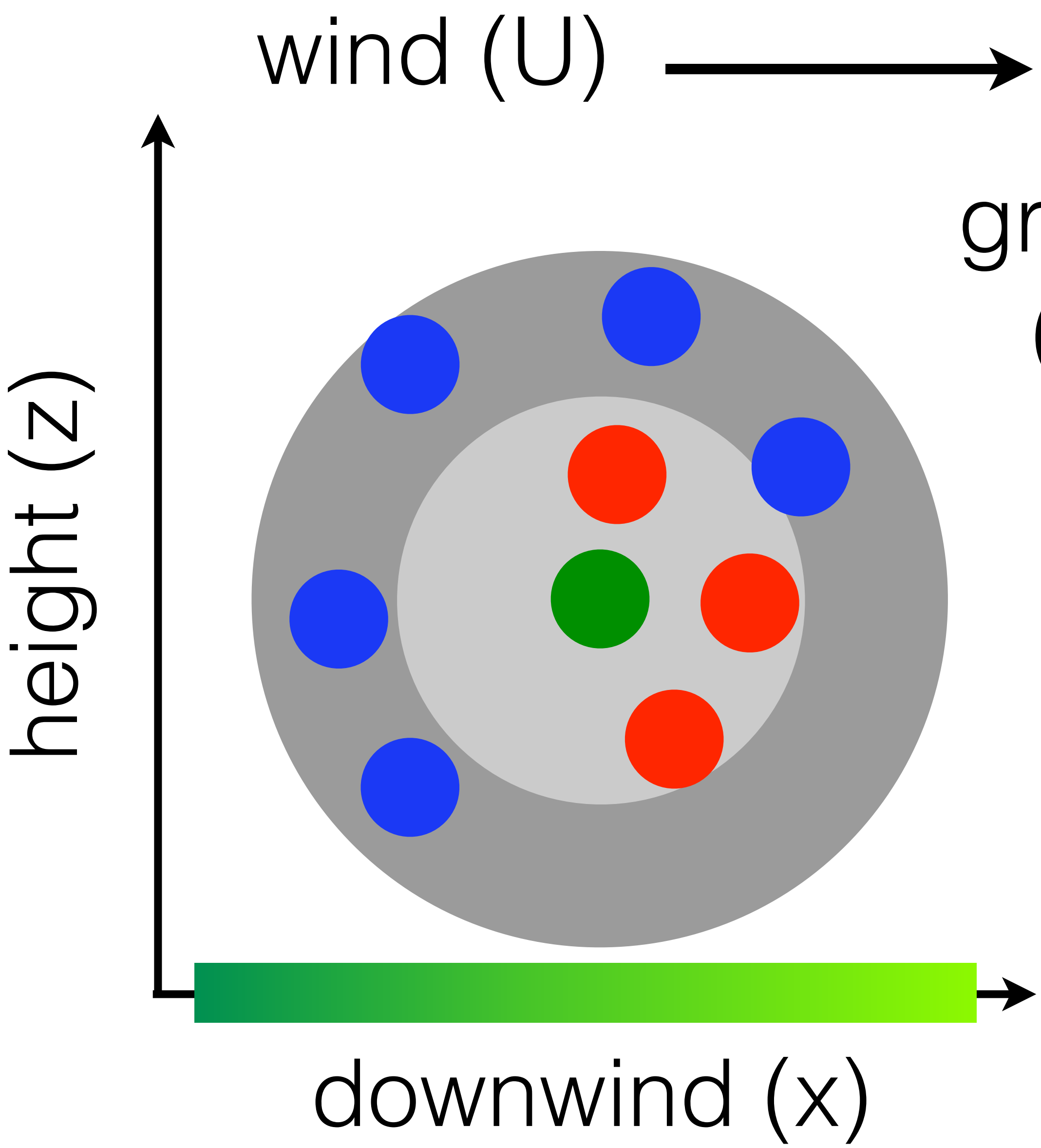
DOI: 10.1140/epjst/e2008-00633-y

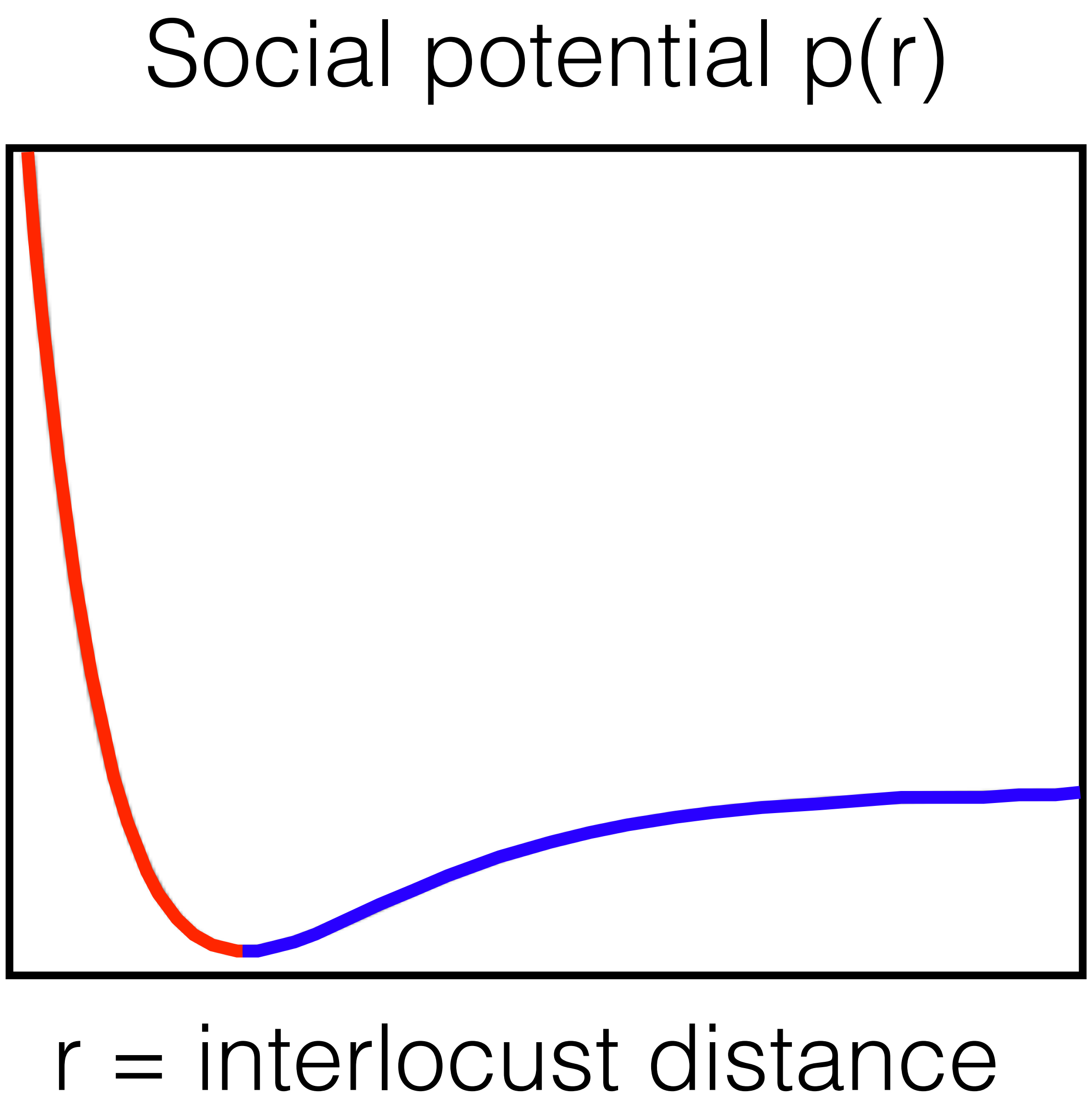
THE EUROPEAN
PHYSICAL JOURNAL
SPECIAL TOPICS

A model for rolling swarms of locusts

C.M. Topaz, A.J. Bernoff, S. Logan, W. Toolson

An agent-based model





Model:

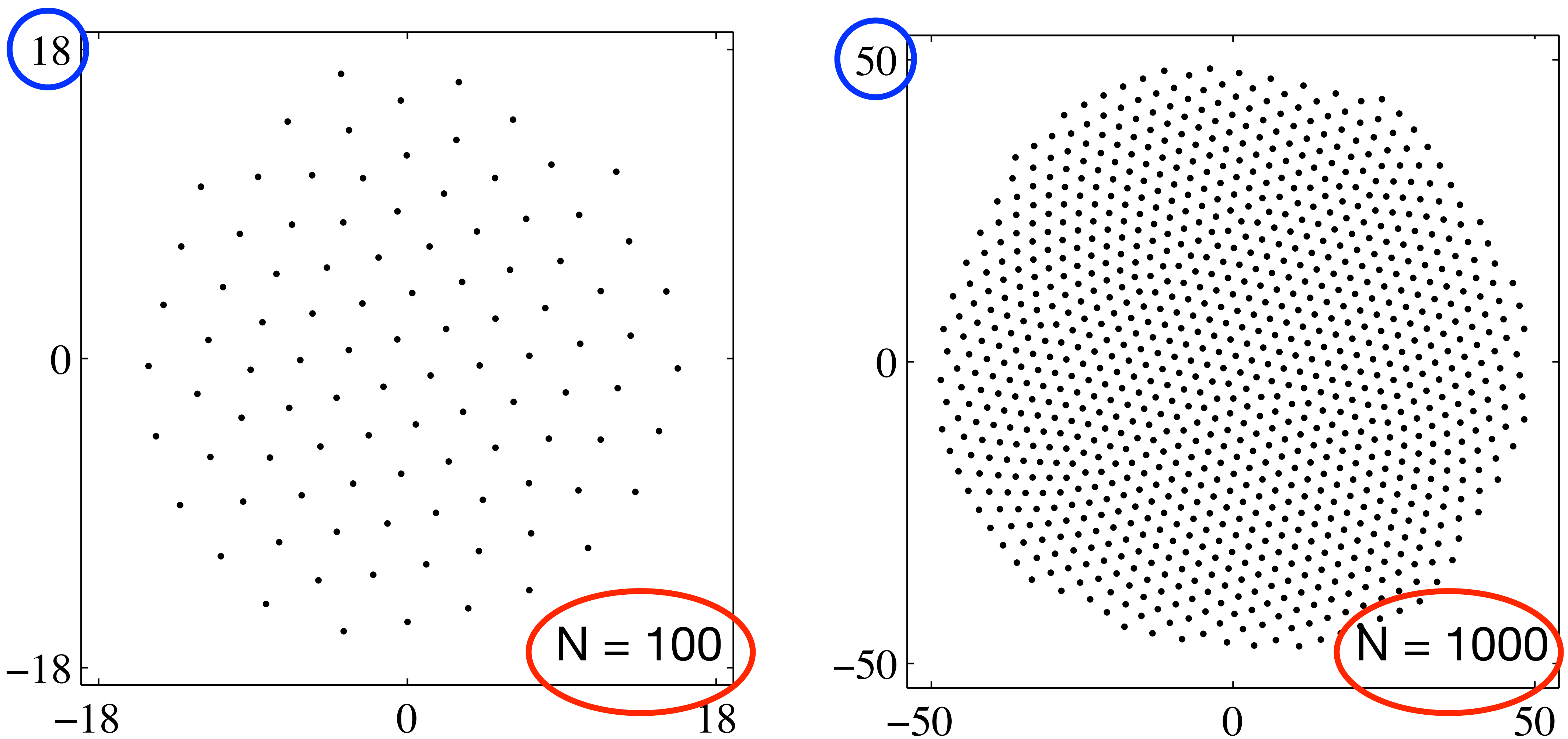
$$\dot{\mathbf{x}}_i = \sum_{j=1}^{N} \frac{dp}{dr}\left(|\mathbf{r}_{ij}|\right)\frac{\mathbf{r}_{ij}}{|\mathbf{r}_{ij}|} - G\widehat{e}_z + U\widehat{e}_x$$

$$p(r) = -FLe^{-r/L} + e^{-r} \quad \mathbf{r}_{ij} = \mathbf{x}_j - \mathbf{x}_i$$

Free-space swarms have two possible behaviors

Behavior #1: H-stable ($FL^3 < 1$)

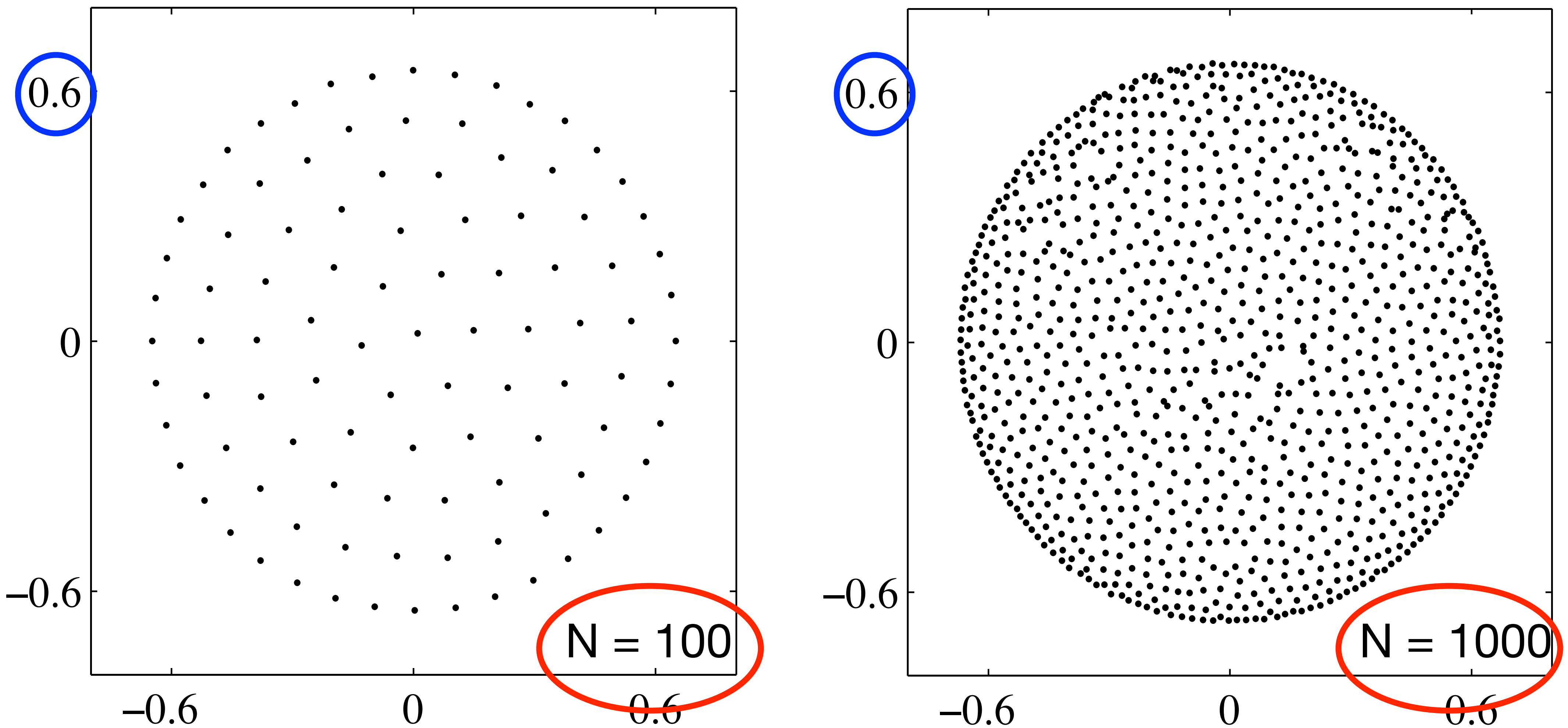
$$p(r) = -FL\mathrm{e}^{-r/L} + \mathrm{e}^{-r}$$



Free-space swarms have two possible behaviors

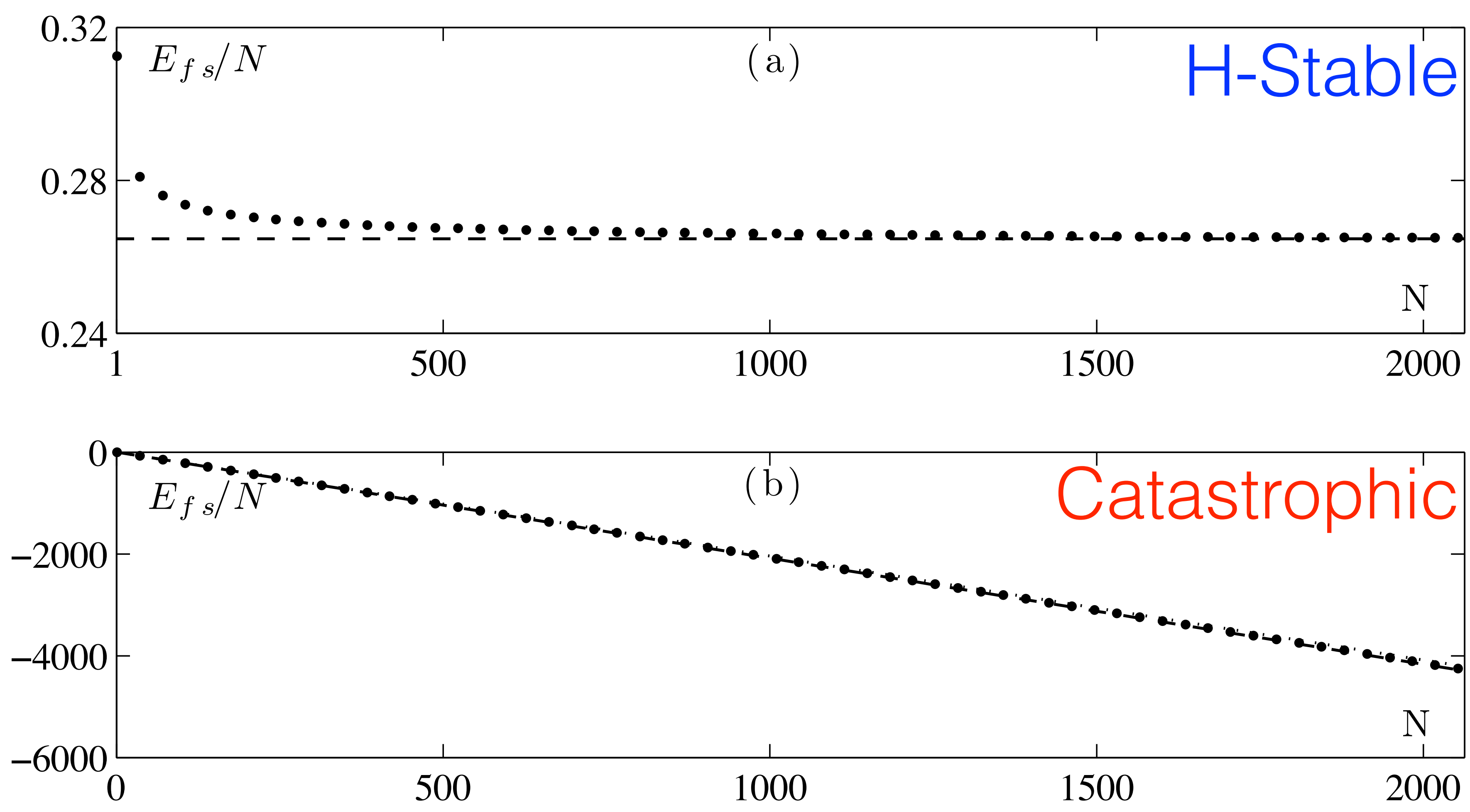
Behavior #2: Catastrophic ($FL^3 > 1$)

$$p(r) = -FL\mathrm{e}^{-r/L} + \mathrm{e}^{-r}$$



Free-space swarms have two possible behaviors

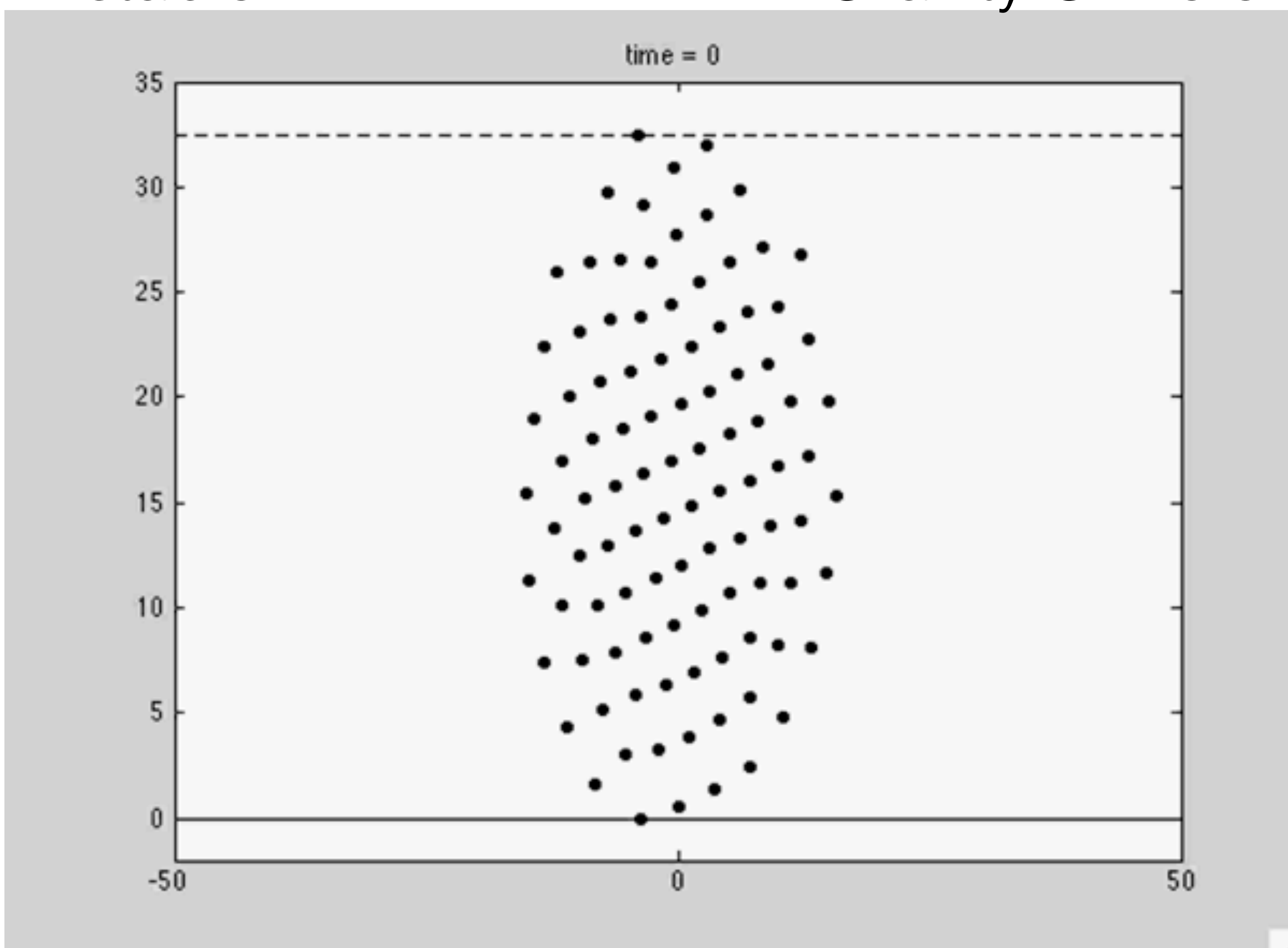
$$E_{fs} = \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} p(r_{ij})$$



With gravity, the catastrophic swarm forms a bubble

H-stable

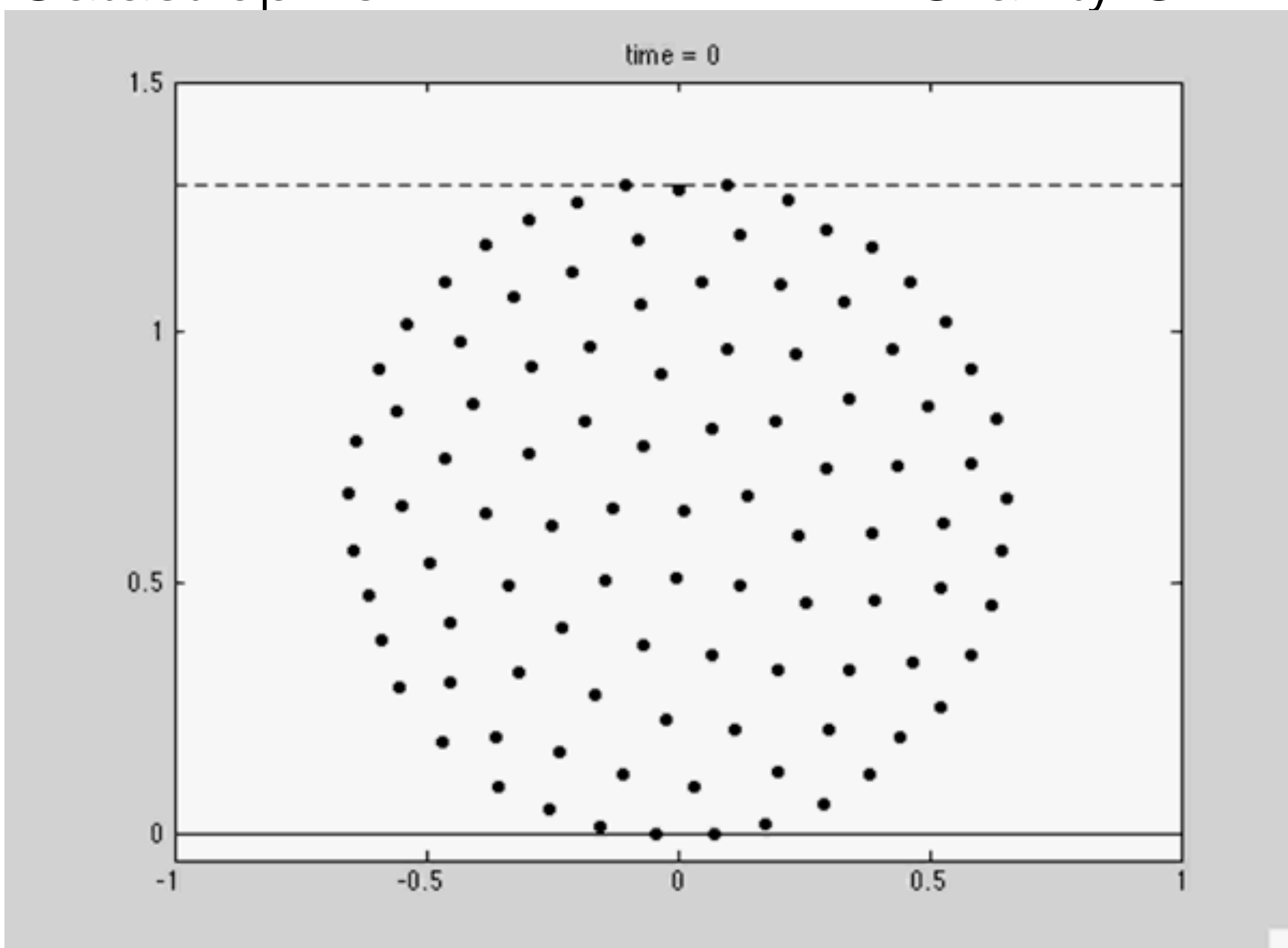
Gravity G = 0.01



With gravity, the catastrophic swarm forms a bubble

Catastrophic

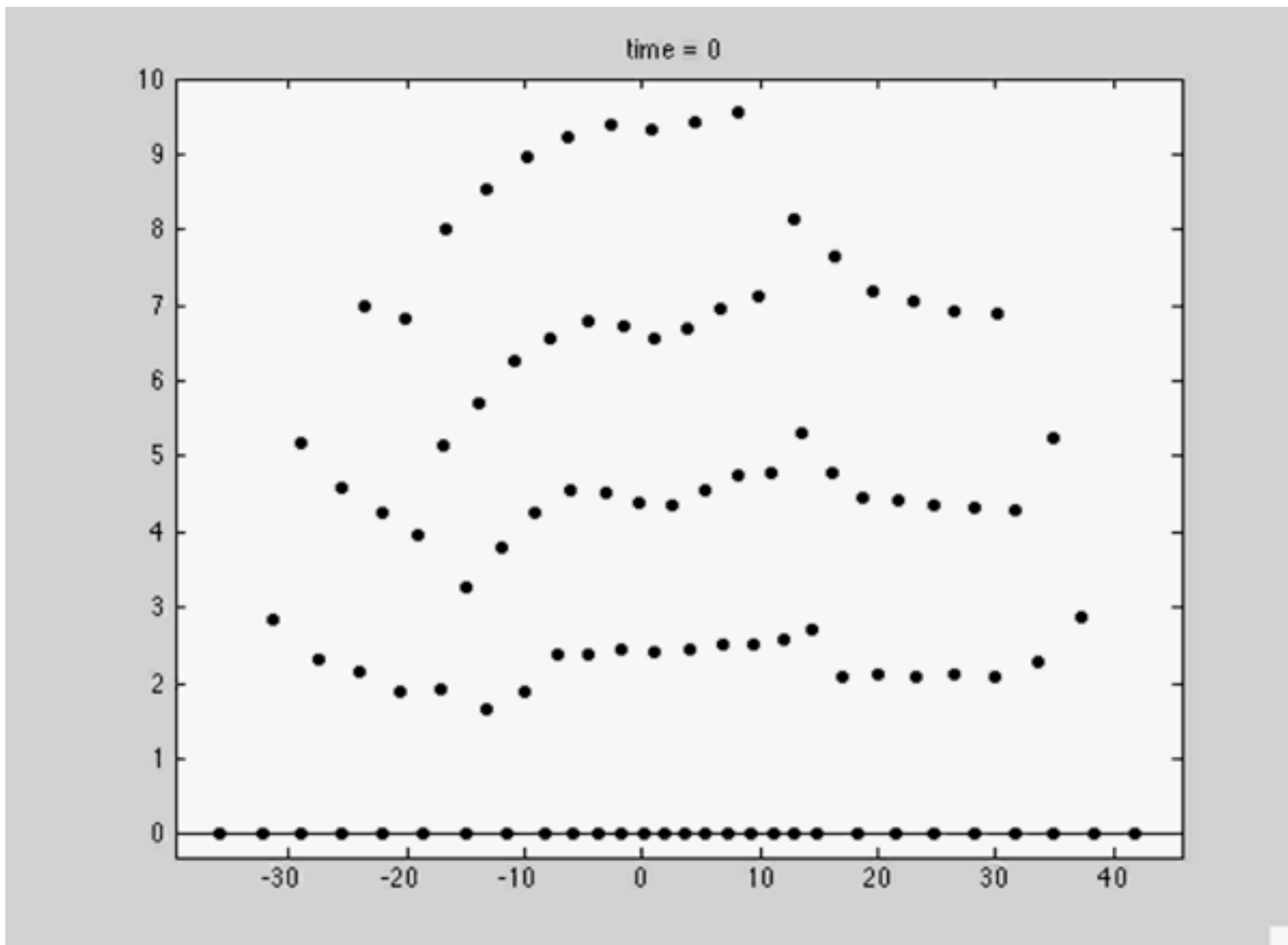
Gravity G = 1



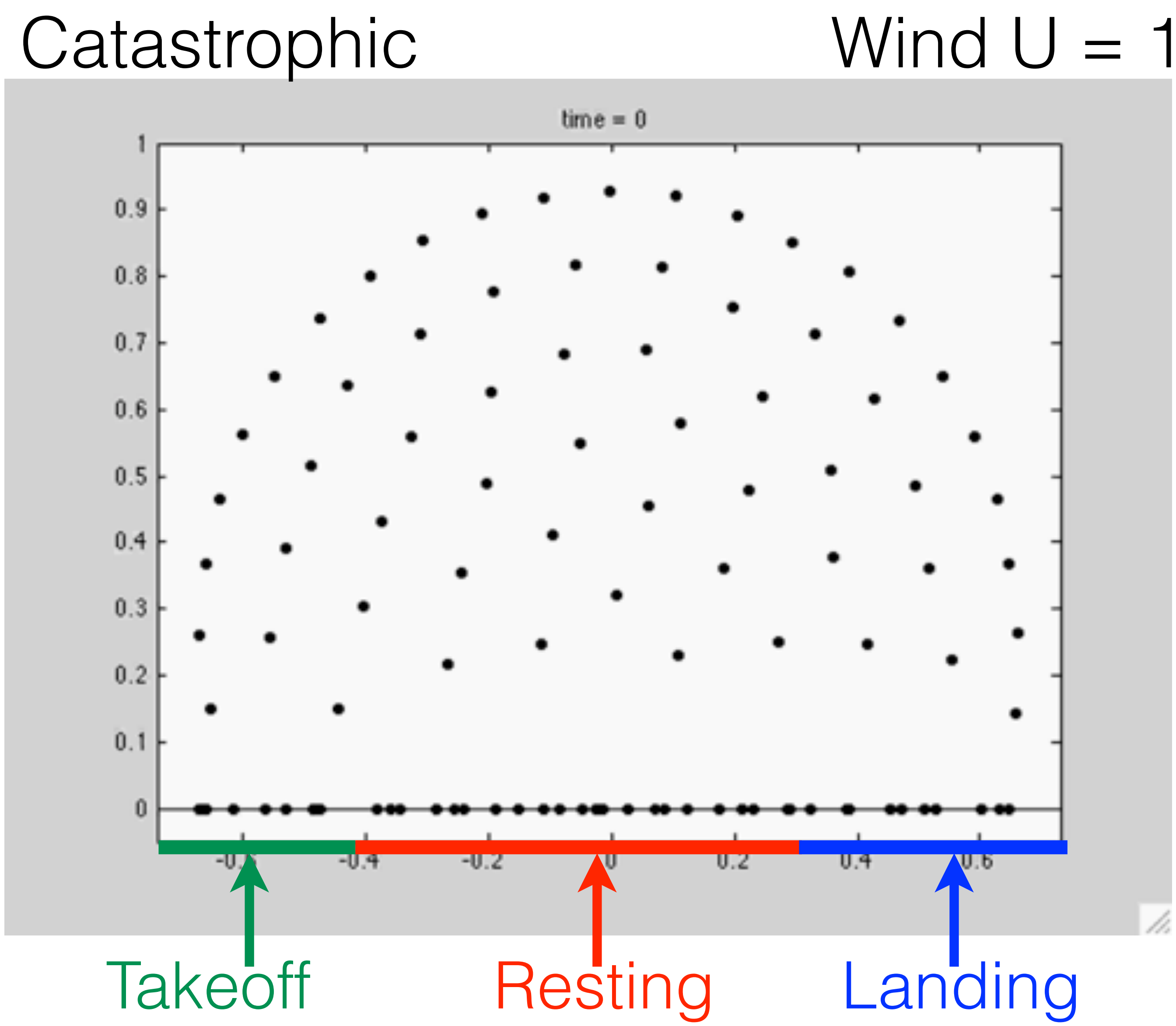
With wind, the H-stable swarm dies out

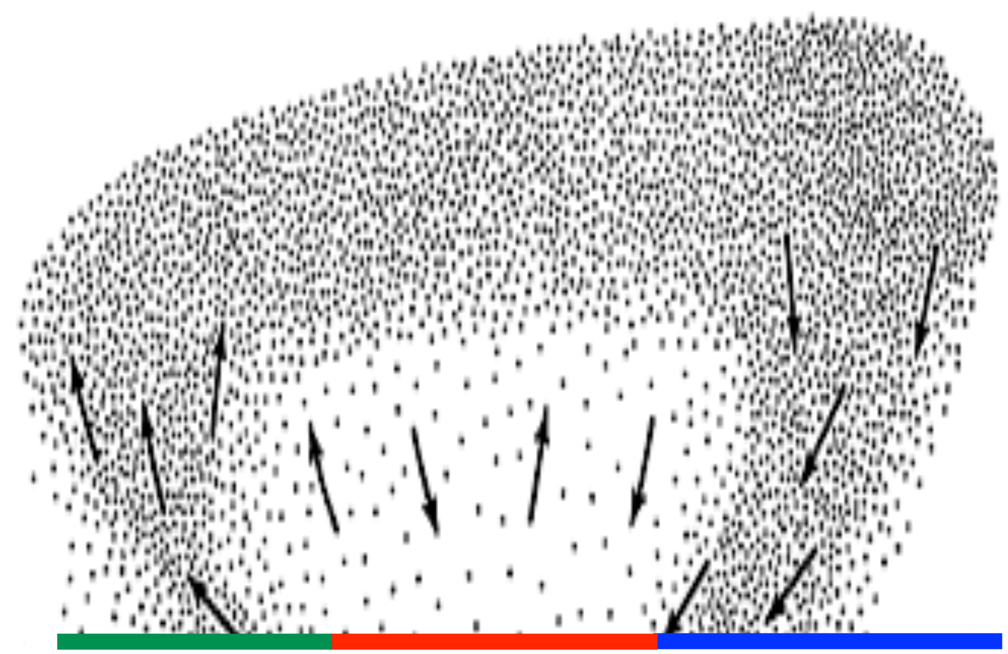
H-Stable

Wind U = 0.01



With wind,
the catastrophic swarm rolls

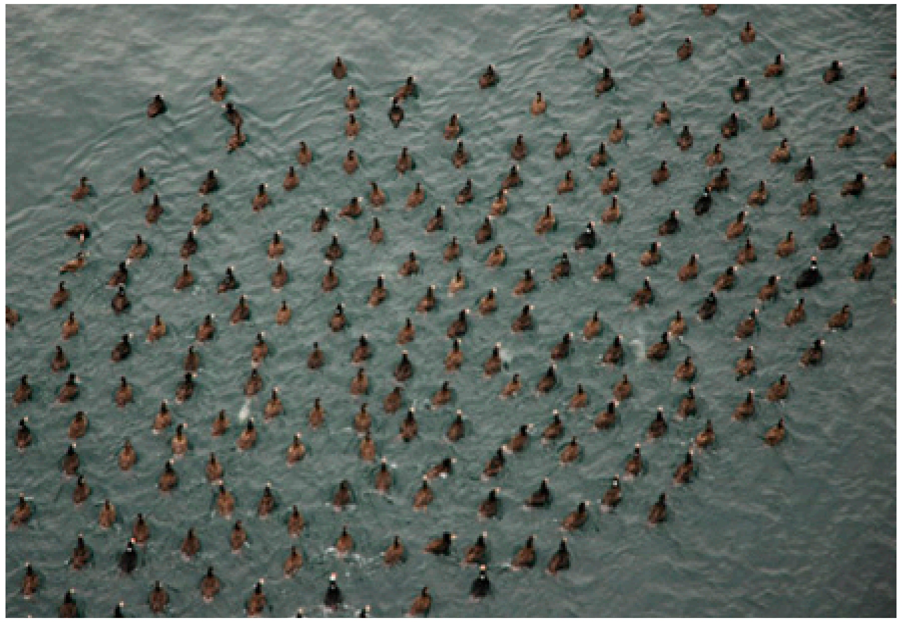




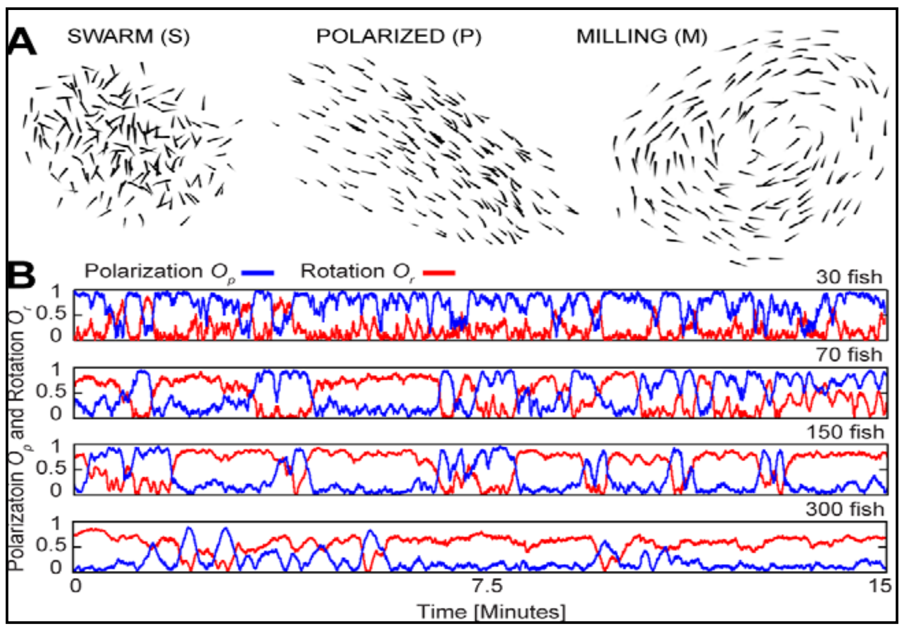
Uvarov, Grasshoppers & Locusts (1977)

Case Study 2: Aphids

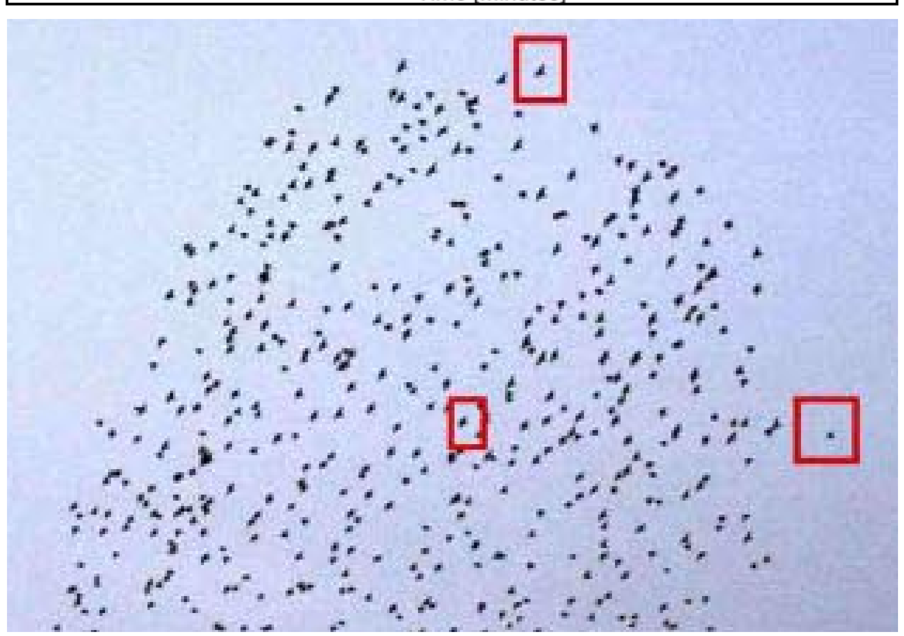
How well are aggregation models tied to data?



Inferring individual rules from collective behavior
R. Lukeman, Y.-X. Li, L. Edelstein-Keshet
PNAS 2010



Collective states, multistability and transitional behavior in schooling fish
Tunstrøm, Katz, *et al.*
PLoS One 2013



Interaction ruling collective animal behavior depends on topological rather than metric distance
M. Ballerini, N. Cabibbo, *et al.*
PNAS 2008

Meet *Acyrthosiphon pisum* (pea aphid)

- Crop pests
- Model organism in biology (disease, phenotypic plasticity, insect-plant interactions...)
- Genomic interest (PLoS Biology, vol. 8, 2010)
- Social aggregators??? (Kidd 1976; Strong 1967)

OPEN ACCESS Freely available online

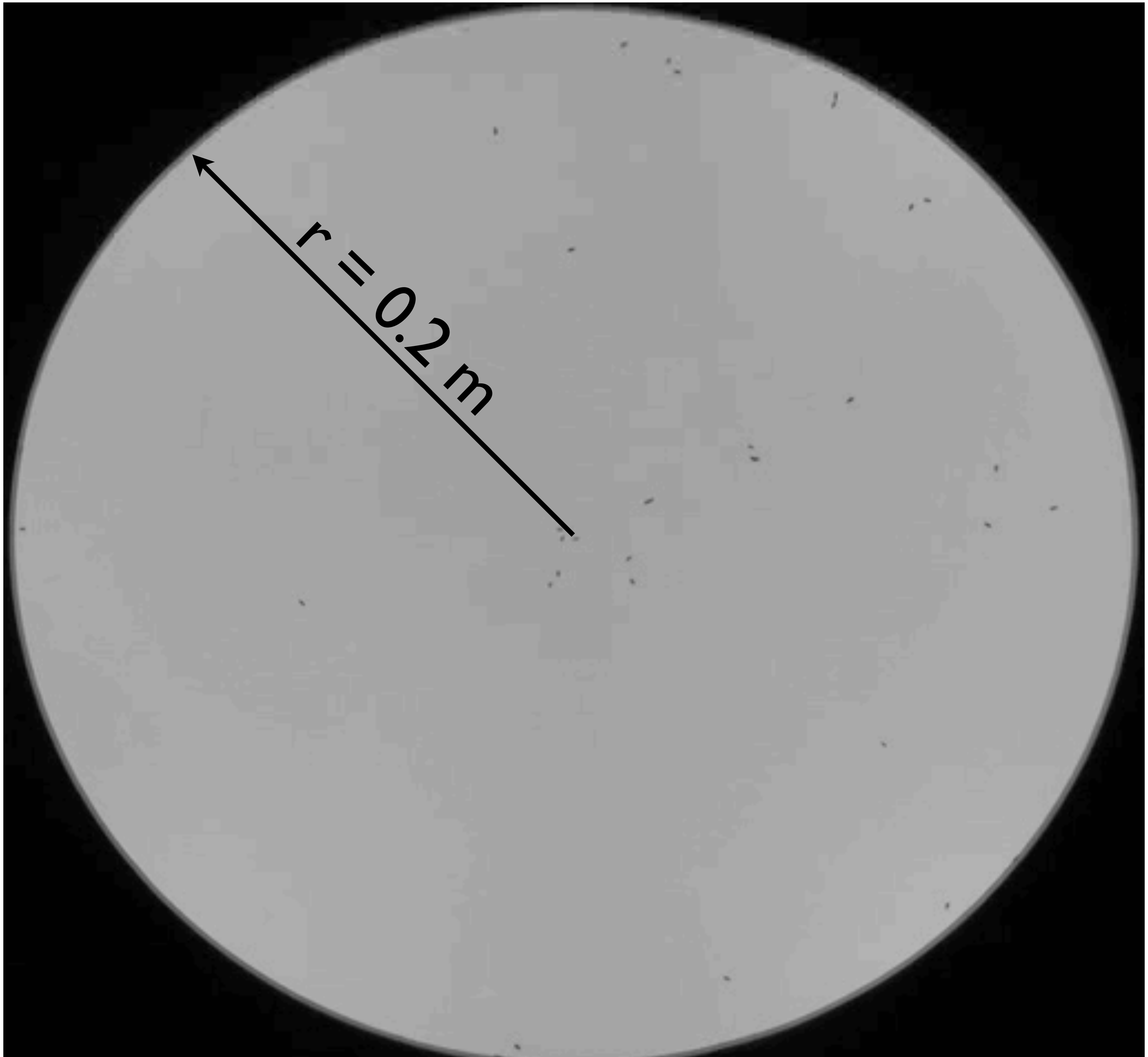
PLOS | ONE

Dec 2013 | Volume 8 | Issue 12 | e83343

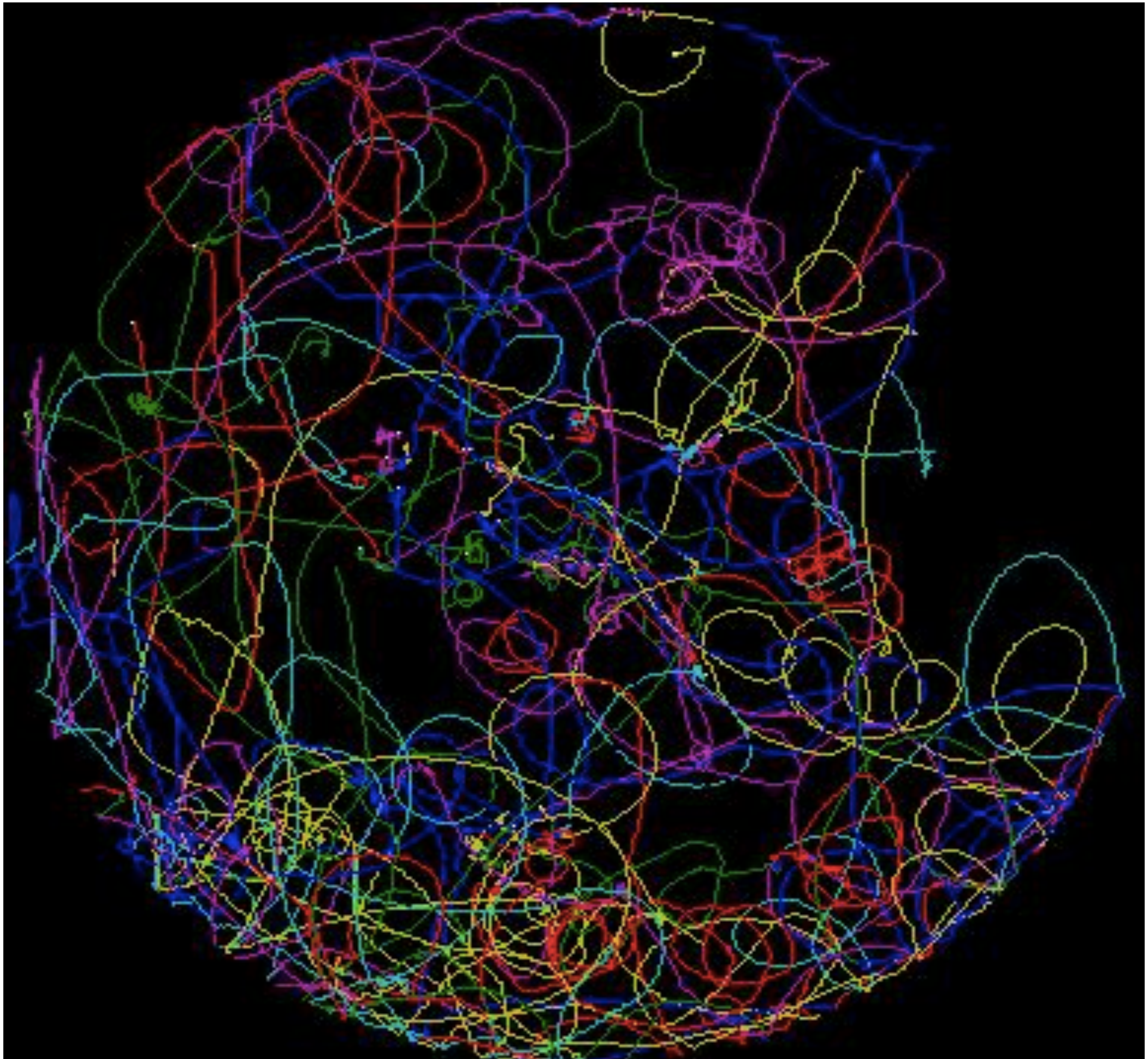
Social aggregation in pea aphids: Experiment and random walk modeling

C. Nilsen*, J. Paige*, O. Warner*, B. Mayhew*, R. Sutley*, M. Lam*, A.J. Bernoff, C.M. Topaz

We filmed pea aphids in a featureless arena.



We applied tracking algorithms to obtain trajectories.



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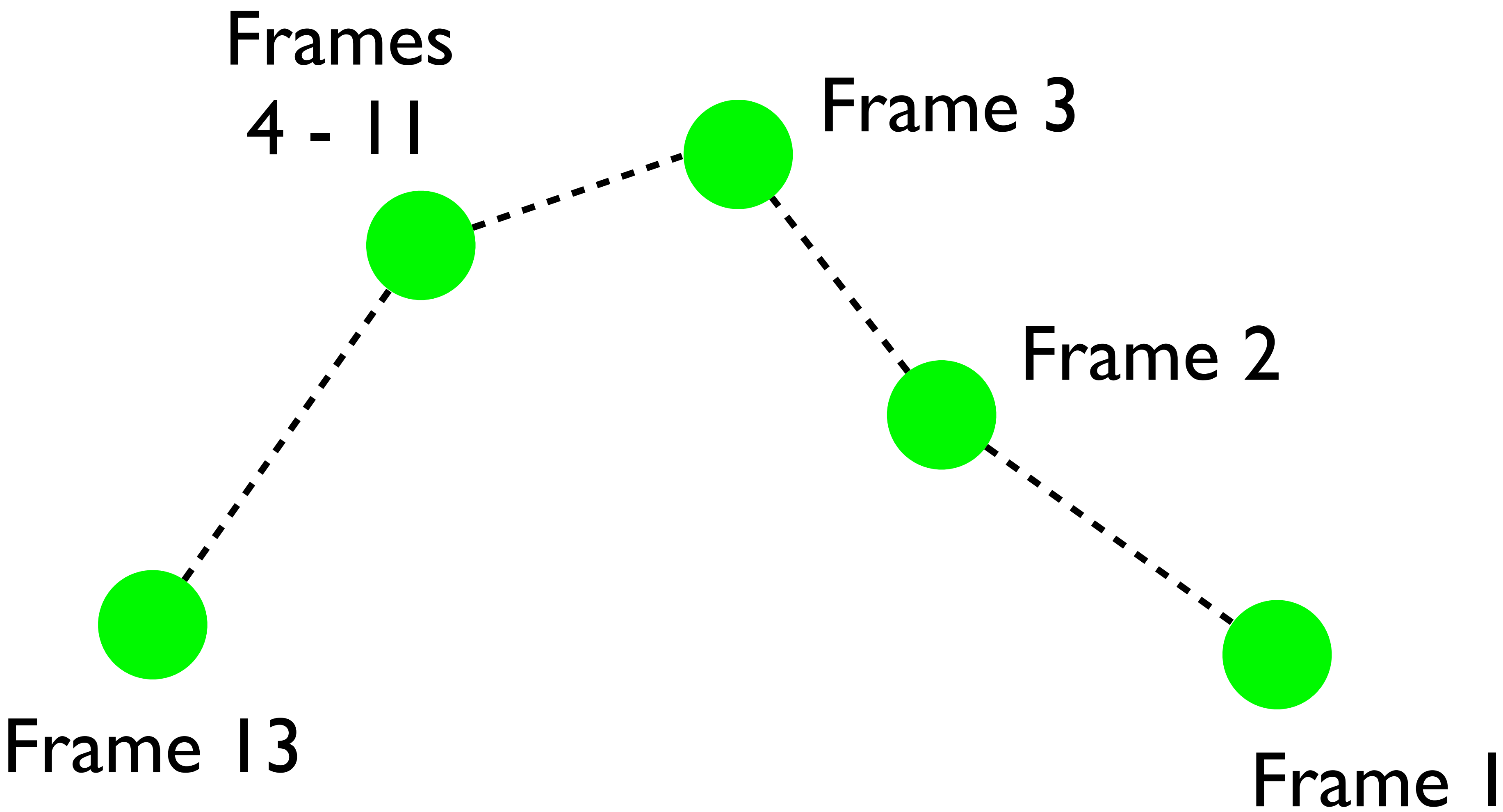
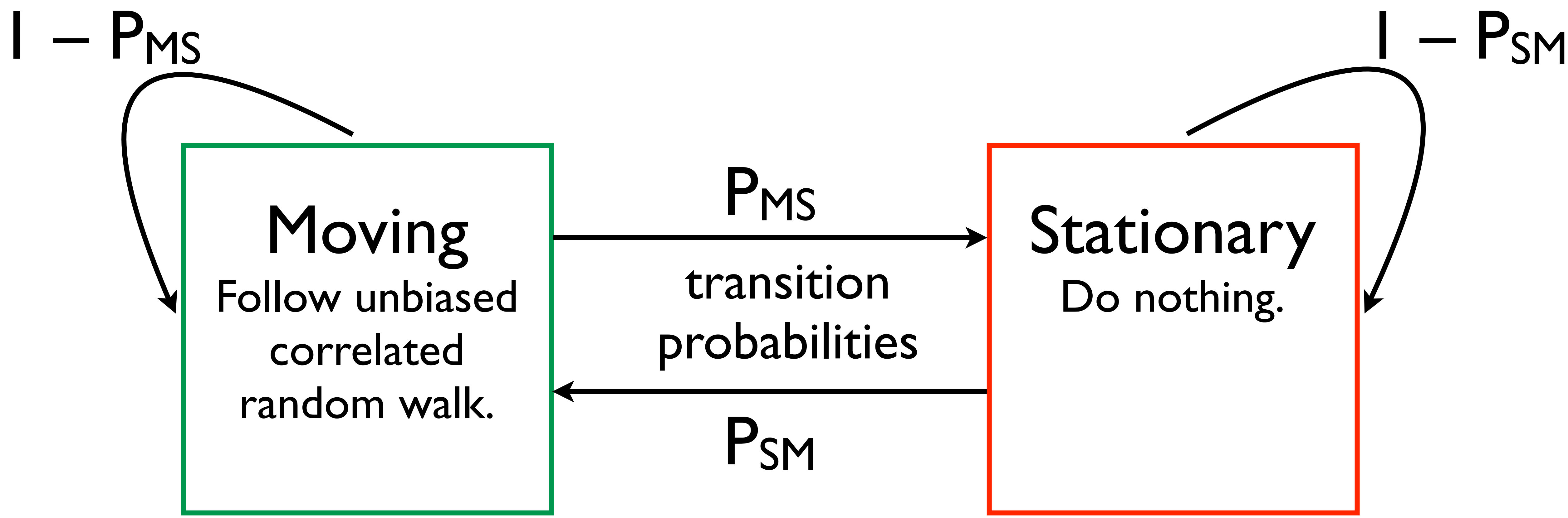
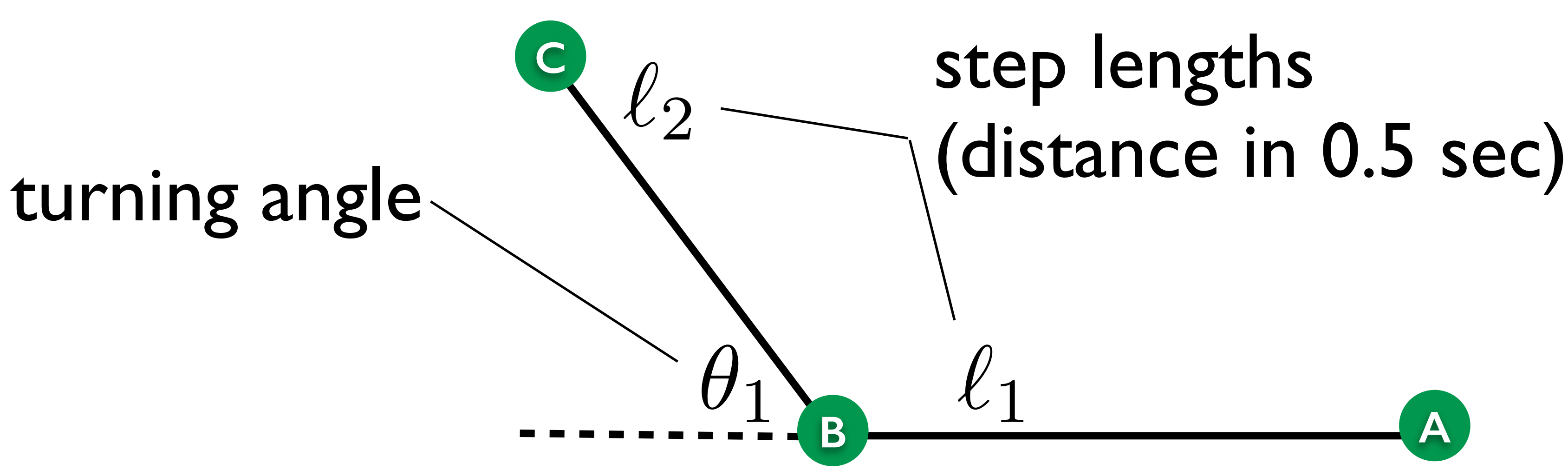


Model it.

We tried a two-state correlated random walk model with four primary parameters.





Analyze data binned by nearest neighbor distance.

Frame #1			
Aphid	**Position**	**Moving?**	**Neighbor Dist.**
1	(#,#)	Y	0.02
2	(#,#)	N	0.06
3	(#,#)	N	0.11
4	(#,#)	Y	0.07

Frame #2			
Aphid	**Position**	**Moving?**	**Neighbor Dist.**
1	(#,#)	Y	0.05
2	(#,#)	Y	0.03
3	(#,#)	N	0.11
4	(#,#)	Y	0.09

Frame #3			
Aphid	**Position**	**Moving?**	**Neighbor Dist.**
1	(#,#)	Y	0.11
2	(#,#)	N	0.03
3	(#,#)	N	0.11
4	(#,#)	N	0.12

Pedagogical example:

- < 0.04 m
- 0.04 – 0.08 m
- 0.08 – 0.12 m

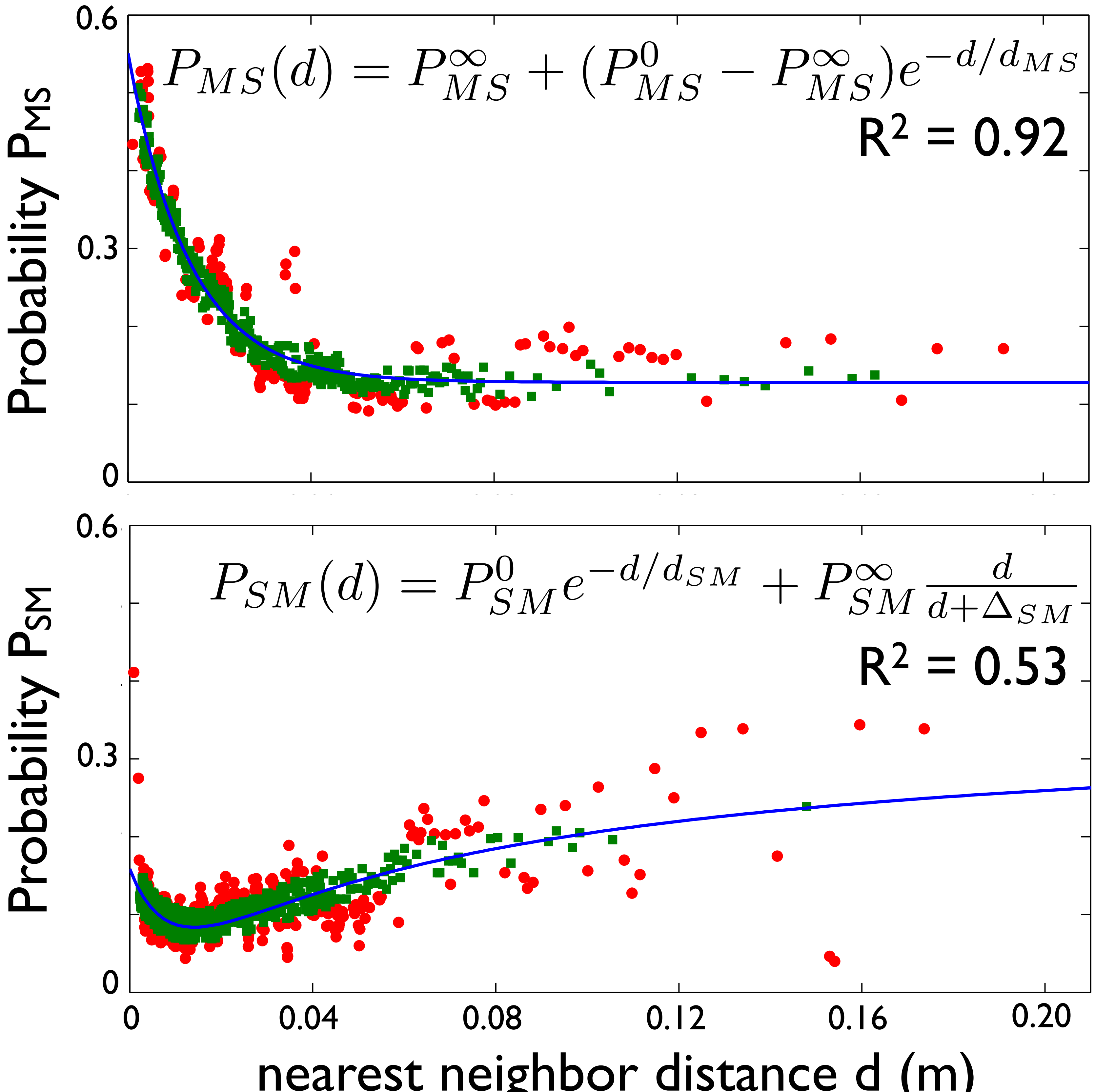
Within each bin, investigate

- Movement transitions
- Step length
- Turning angle

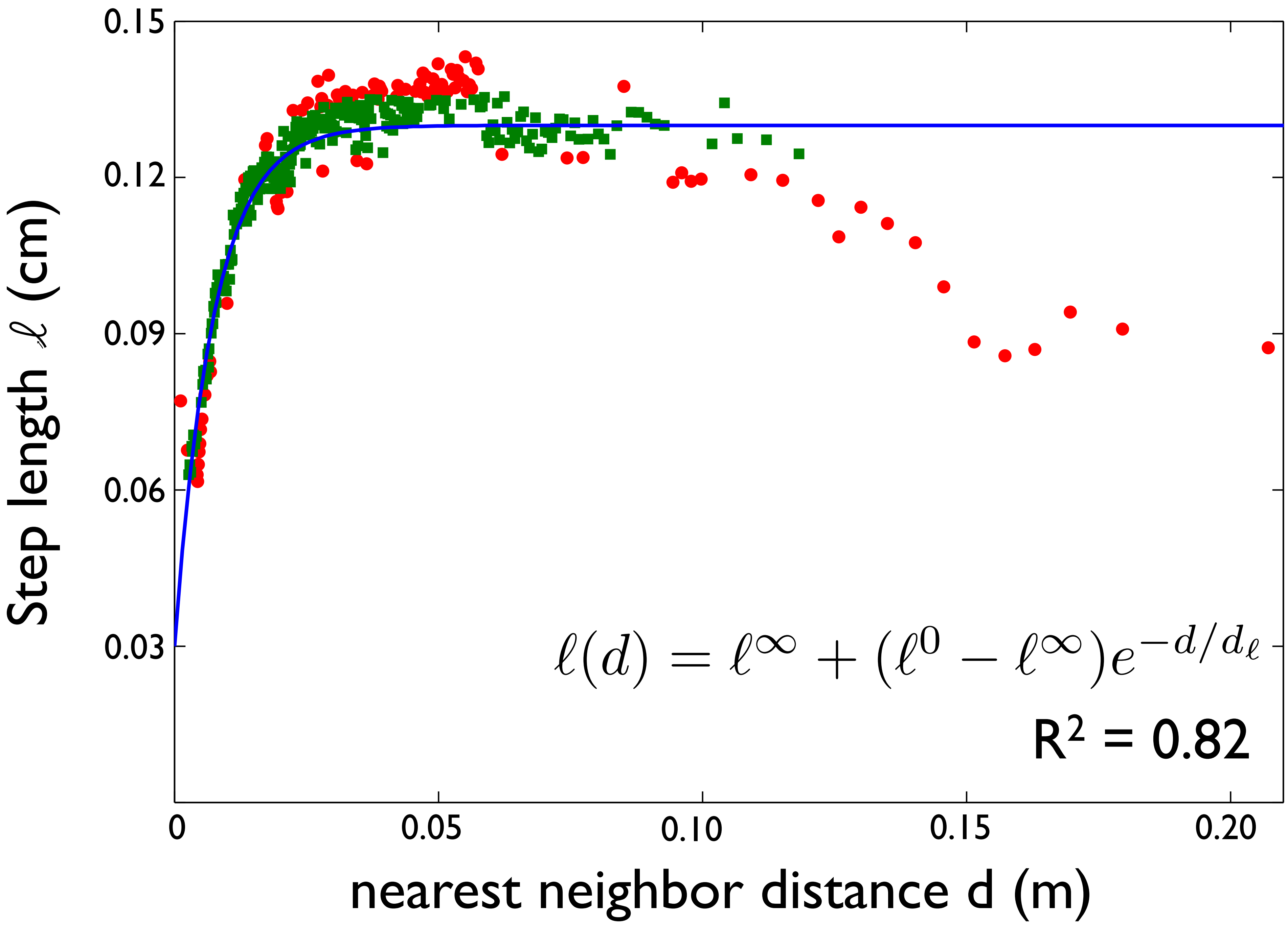
In the real data

- 1 bin ≈ 800 observations
- 1.2 million observations

State transition probabilities depend on distance to an aphid's nearest neighbor.

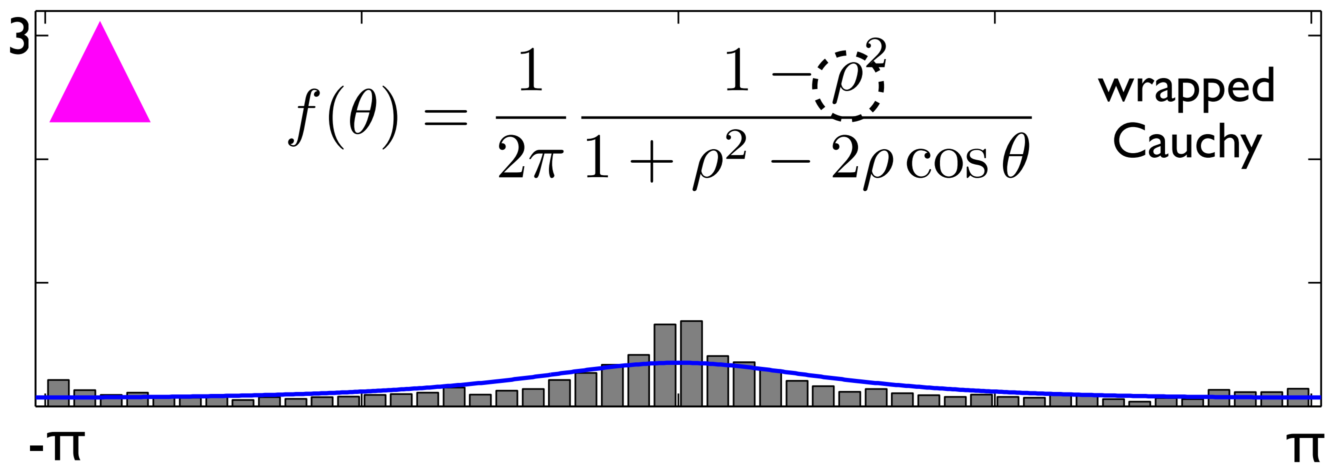


Step lengths depend on distance to an aphid's nearest neighbor.

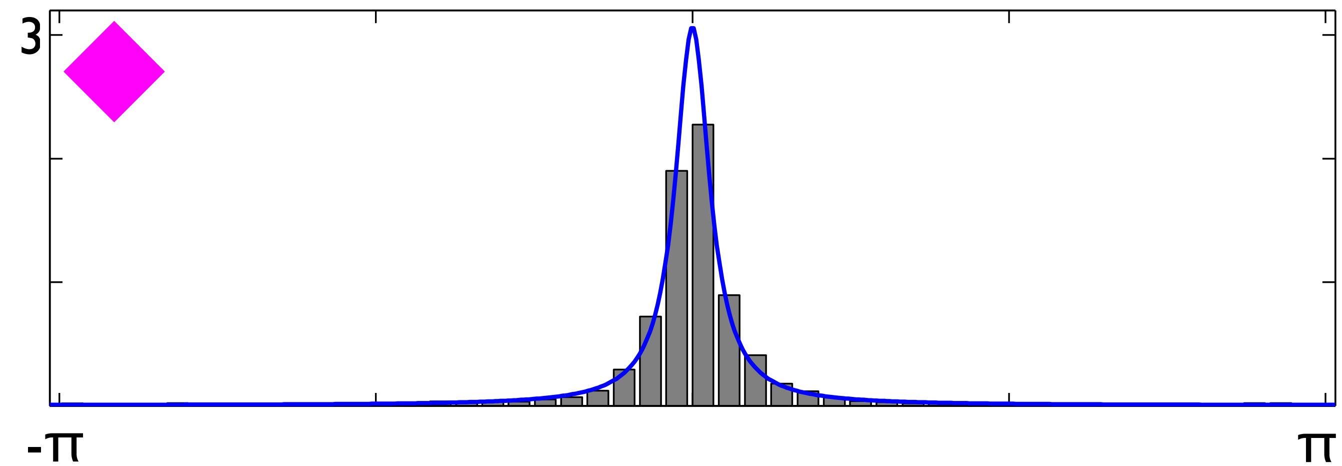


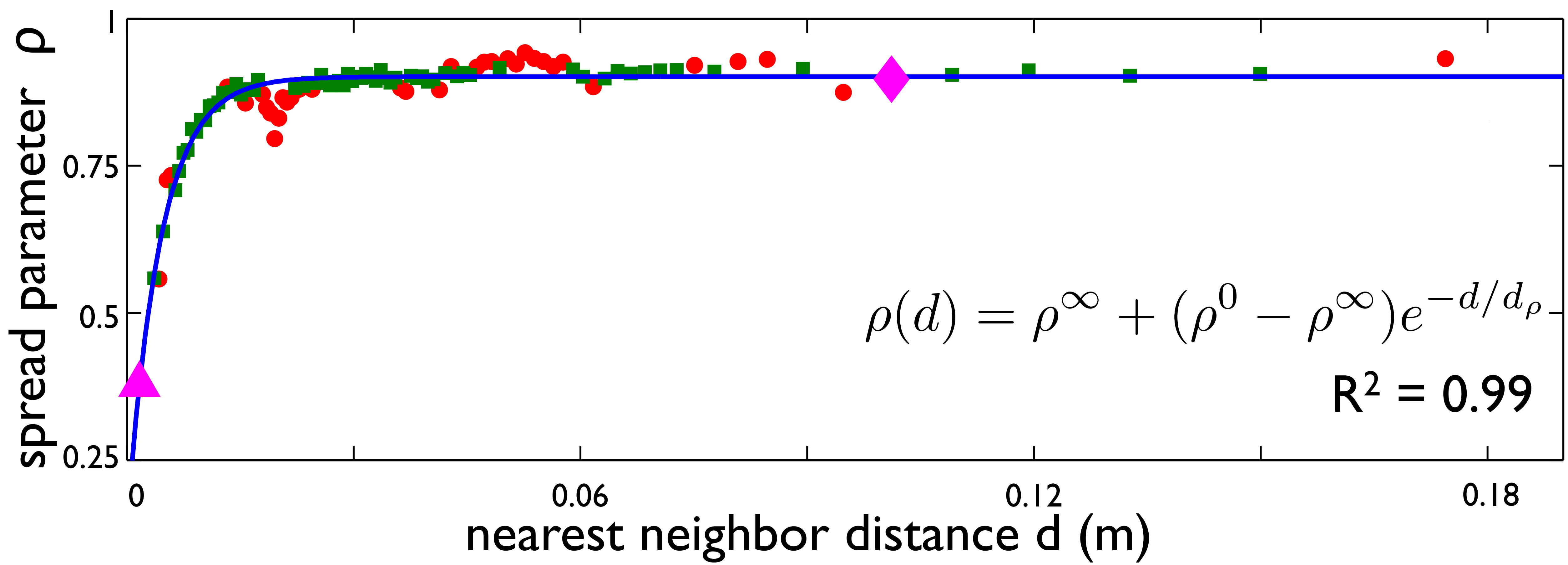
Turning angle distribution spread depends on distance to an aphid's nearest neighbor.

Turning angle distribution

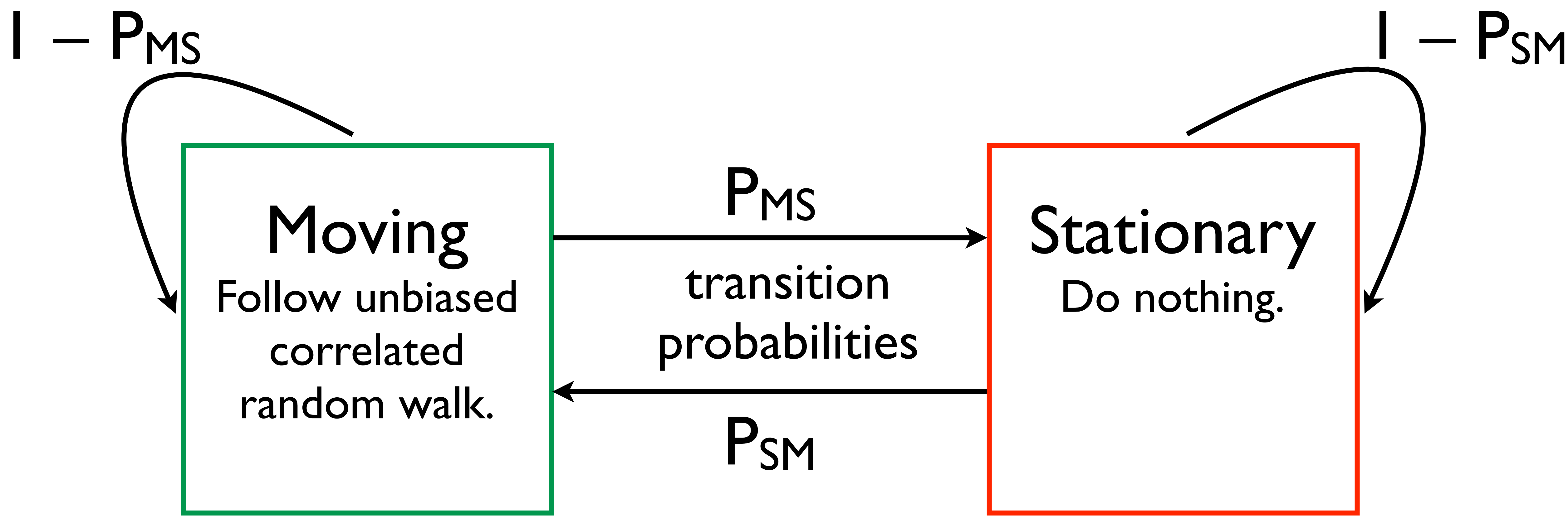


Turning angle distribution





In summary of the model...



Estimated from a data set of 1.2 million entries:

Quantity	Meaning	Depends on nearest neighbor distance d via...
P_{MS}	Probability of stopping	3 parameters
P_{SM}	Probability of starting	4 parameters
ℓ	Step length	3 parameters
ρ	Turning angle spread	3 parameters

Main messages:

- Sensing range 0.4 - 1.2 cm (1 - 3 body lengths)
- Lonely: P(move) ↑, step length ↑, turning angle ↓
- Crowded: P(move) ↓, step length ↓, turning angle ↑
- Social behavior, passive aggregation mechanism
- Social model gives better agreement with experiment

Conclusions

1. There are many agent based aggregation models.

2. Some have been influential.

3. Choose the right model for the question(s) you want to answer.

4. Actually, please, just HAVE a question that you want to answer.

The big picture

- Questions about biological aggregations

    1. How does each individual behave?

    2. How does the group behave?

    3. How are individual and group behavior linked?