The Discrete Conceit: Agent-Based Aggregation Models

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The big picture

- Biological aggregations are social groups displaying collective behavior.

- They matter for pattern formation, biology, algorithms…

- Consider a minimal modeling approach.

- There are hundreds or thousands of published models, and some have been highly influential.

- Choose the right model for the job.
Biological Aggregations
Biological aggregations move in a coordinated manner.

Parrish & Keshet, Nature, 1999
Aggregations can propagate without a leader.
Social interactions are key to the formation of groups.
Social interactions are key to the formation of groups.
Who cares?

“Social behaviors [that] on short time and space scales lead to the formation and maintenance of groups... lead at larger time and space scales to differences in spatial distributions of populations and rates of encounter and interaction with populations of predators, prey, competitors and pathogens... At the largest time and space scales, aggregation has profound consequences for ecosystem dynamics and for evolution of behavioral, morphological, and life history traits.”

Collective motion occurs across the natural and engineered worlds.
Aggregation Models
There are numerous classes of aggregation models:

- Continuum
- Discrete
- Kinematic
- Dynamic
- Deterministic
- Stochastic
- Isotropic
- Nonisotropic
Consider the value of parsimony in your models.

Fish neurobiology
Fish behavior
Ocean current profiles
Fluid dynamics
Resource distribution

Self-propulsion
Attraction/repulsion
Alignment
Example 1: Attraction/repulsion

[PDF] Equations descriptive of fish schools and other animal aggregations
CM Breder - Ecology, 1954 - JSTOR
In an effort to understand better the basic nature of the influences at work in a school of fishes as well as in other less compact aggregations, Breder (1951) discussed, in passing, the possibility of applying physical equations to such groups, without going into the matter ...

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His earliest publication, written at 18, concerned photography of local birds and by age 21 he had published 15 popular articles and notes and had started his theoretical and experimental studies on fish locomotion. In 1925 the New York Academy of Sciences awarded him the A. Cressy Morrison Prize for his pioneering and penetrating analysis of fish locomotion...
Example 1: Attraction/repulsion

\[ c = \frac{a}{d^m} - \frac{r}{d^n} \]

attraction

force between organisms/groups

repulsion
distance

m = 0, n = 2

force c

distance d
Novel type of phase transition in a system of self-driven particles

T Vicsek, A Czirók, E Ben-Jacob, I Cohen, O Shochet - Physical review letters, 1995 - APS

Abstract A simple model with a novel type of dynamics is introduced in order to investigate the emergence of self-ordered motion in systems of particles with biologically motivated interaction. In our model particles are driven with a constant absolute velocity and at each ...

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Example 2: Kinematic Alignment

Averaging + Noise
Example 2: Kinematic Alignment

\[
\theta_i \leftarrow \frac{1}{N} \sum_{|x_i - x_j| \leq R} \theta_j + U(-\eta/2, \eta/2)
\]

\[
v_i \leftarrow v_0(\cos \theta_i, \sin \theta_i)
\]

\[
x_i \leftarrow x_i + v_i \Delta t
\]

http://youtu.be/jphRZV3oCaI
Example 2: Kinematic Alignment

Clusters
Loose alignment
Strong alignment
Example 3: Dynamic, Self-Propelled Swarmers
Example 3: Dynamic, Self-Propelled Swarmers

\[
\begin{align*}
\dot{x}_i &= v_i \\
m\dot{v}_i &= (\alpha - \beta |v_i|^2) v_i - \nabla_i Q_i \\
Q_i &= \sum_{j \neq i} C_r e^{-|x_i - x_j|/L_r} \\
&\quad - C_a e^{-|x_i - x_j|/L_a}.
\end{align*}
\]
Example 3: Dynamic, Self-Propelled Swarmers

Social potential $Q(r)$

$r = \text{interorganism distance}$
Example 3: Dynamic, Self-Propelled Swarmers

\[
\dot{x}_i = v_i
\]

\[
m\ddot{v}_i = \left(\alpha - \beta |v_i|^2\right) v_i - \nabla_i Q_i
\]

\[
Q_i = \sum_{j \neq i} C_r e^{-|x_i - x_j|/L_r} - C_a e^{-|x_i - x_j|/L_a}
\]
Example 4:
Kitchen Sink

Collective memory and spatial sorting in animal groups
ID Couzin, J Krause, R James, GD Ruxton… - Journal of theoretical …, 2002 - Elsevier
We present a self-organizing model of group formation in three-dimensional space, and use it to investigate the spatial dynamics of animal groups such as fish schools and bird flocks. We reveal the existence of major group-level behavioural transitions related to minor ...
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Case Study 1: Locusts
Locusts = devastating

- Winter/spring breeding areas and resulting migration
- Summer breeding areas and resulting migration

Desert Locust Range

10^{10} locusts

100 km^2

10 - 100 km/day
Locust swarms migrate with a rolling motion

Uvarov, *Grasshoppers and Locusts* (1977)
Model it.
A model for rolling swarms of locusts
C.M. Topaz, A.J. Bernoff, S. Logan, W. Toolson
Model:

\[ \dot{x}_i = \sum_{j=1}^{N} \frac{dp}{dr} \left( |r_{ij}| \right) \frac{r_{ij}}{|r_{ij}|} - G \hat{e}_z + U \hat{e}_x \]

\[ p(r) = -F L e^{-r/L} + e^{-r} \]

\[ r_{ij} = x_j - x_i \]
Free-space swarms have two possible behaviors

Behavior #1: H-stable ($FL^3 < 1$)

\[ \dot{x}_i = -\nabla_i E_{fs}, \quad E_{fs} = \frac{1}{2} N \sum_{i=1}^{N} \sum_{j=1}^{N} p(r_{ij}) \]

\[ p(r) = -FLe^{-r/L} + e^{-r} \]

---

**Diagram:**
- Left: $N = 100$
- Right: $N = 1000$
Free-space swarms have two possible behaviors

Behavior #2: Catastrophic ($FL^3 > 1$)

\[ p(r) = -F L e^{-r/L} + e^{-r} \]
Free-space swarms have two possible behaviors

\[ E_{fs} = \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} p(r_{ij}) \]
With gravity, the catastrophic swarm forms a bubble

H-stable

Gravity G = 0.01
With gravity, the catastrophic swarm forms a bubble

Catastrophic Gravity $G = 1$
With wind, the H-stable swarm dies out

H-Stable

Wind U = 0.01
With wind, the catastrophic swarm rolls

Uvarov, Grasshoppers & Locusts (1977)
Case Study 2: Aphids
How well are aggregation models tied to data?

Inferring individual rules from collective behavior
R. Lukeman, Y.-X. Li, L. Edelstein-Keshet
PNAS 2010

Collective states, multistability and transitional behavior in schooling fish
Tunstrøm, Katz, et al.
PLoS One 2013

Interaction ruling collective animal behavior depends on topological rather than metric distance
M. Ballerini, N. Cabibbo, et al.
PNAS 2008
Meet *Acyrthosiphon pisum* (pea aphid)

- Crop pests
- Model organism in biology (disease, phenotypic plasticity, insect-plant interactions...)
- Social aggregators?? (Kidd 1976; Strong 1967)
Social aggregation in pea aphids: Experiment and random walk modeling

We filmed pea aphids in a featureless arena.

\[ r = 0.2 \text{ m} \]
We applied tracking algorithms to obtain trajectories.
We applied tracking algorithms to obtain trajectories.
Model it.
We tried a two-state correlated random walk model with four primary parameters.

I - P_{MS}  \quad \text{Moving} \quad \text{Follow unbiased correlated random walk.}

P_{MS} \quad \text{transition probabilities}

P_{SM}  \quad \text{Stationary} \quad \text{Do nothing.}

I - P_{SM}  

\text{turning angle}

\theta_1

\ell_1

\text{step lengths (distance in 0.5 sec)}

\ell_2

C

B

A
Analyze data binned by nearest neighbor distance.

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Pedagogical example:
- < 0.04 m
- 0.04 – 0.08 m
- 0.08 – 0.12 m

Within each bin, investigate:
- Movement transitions
- Step length
- Turning angle

In the real data:
- 1 bin ≈ 800 observations
- 1.2 million observations
State transition probabilities depend on distance to an aphid’s nearest neighbor.

\[ P_{MS}(d) = P_{MS}^\infty + (P_{MS}^0 - P_{MS}^\infty)e^{-d/d_{MS}} \]

\[ R^2 = 0.92 \]

\[ P_{SM}(d) = P_{SM}^0 e^{-d/d_{SM}} + P_{SM}^\infty \frac{d}{d + \Delta_{SM}} \]

\[ R^2 = 0.53 \]
Step lengths depend on distance to an aphid’s nearest neighbor.

\[ \ell(d) = \ell_\infty + (\ell^0 - \ell_\infty) e^{-d/d_\ell} \]

\[ R^2 = 0.82 \]
Turning angle distribution spread depends on distance to an aphid’s nearest neighbor.

\[ f(\theta) = \frac{1}{2\pi} \frac{1 - \rho^2}{1 + \rho^2 - 2\rho \cos \theta} \]

\[ \rho(d) = \rho^\infty + (\rho^0 - \rho^\infty) e^{-d/d\rho} \]

\[ R^2 = 0.99 \]
In summary of the model...

- **Moving**
  - Follow unbiased correlated random walk.
  - $1 - P_{MS}$

- **Stationary**
  - Do nothing.
  - $1 - P_{SM}$

Transition probabilities:

- $P_{MS}$
- $P_{SM}$

Estimated from a data set of 1.2 million entries:

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<th>Quantity</th>
<th>Meaning</th>
<th>Depends on nearest neighbor distance $d$ via...</th>
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<td>$P_{MS}$</td>
<td>Probability of stopping</td>
<td>3 parameters</td>
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<tr>
<td>$P_{SM}$</td>
<td>Probability of starting</td>
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<tr>
<td>$\lambda$</td>
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<tr>
<td>$\rho$</td>
<td>Turning angle spread</td>
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Main messages:

• Sensing range 0.4 - 1.2 cm (1 - 3 body lengths)
• Lonely: $P(\text{move}) \uparrow$, step length $\uparrow$, turning angle $\downarrow$
• Crowded: $P(\text{move}) \downarrow$, step length $\downarrow$, turning angle $\uparrow$
• Social behavior, passive aggregation mechanism
• Social model gives better agreement with experiment
Conclusions

1. There are many agent based aggregation models.

2. Some have been influential.

3. Choose the right model for the question(s) you want to answer.

4. Actually, please, just HAVE a question that you want to answer.
The big picture

• Questions about biological aggregations
  1. How does each individual behave?
  2. How does the group behave?
  3. How are individual and group behavior linked?