The Mathematics of Criminal Behavior: From Agent-Based Models to PDEs

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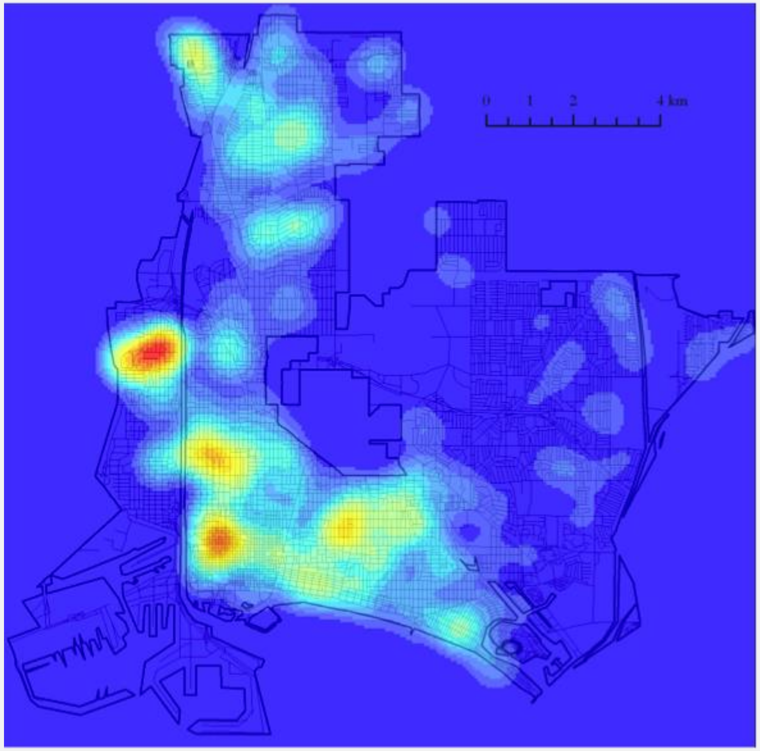
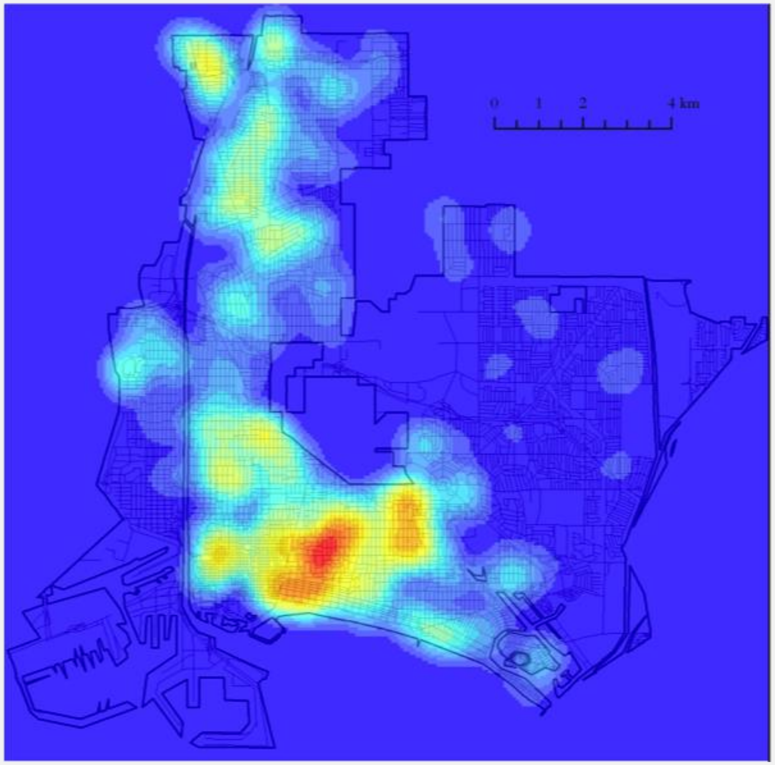
First things first: crime exhibits spatial and/or temporal patterning.

April-June 2001



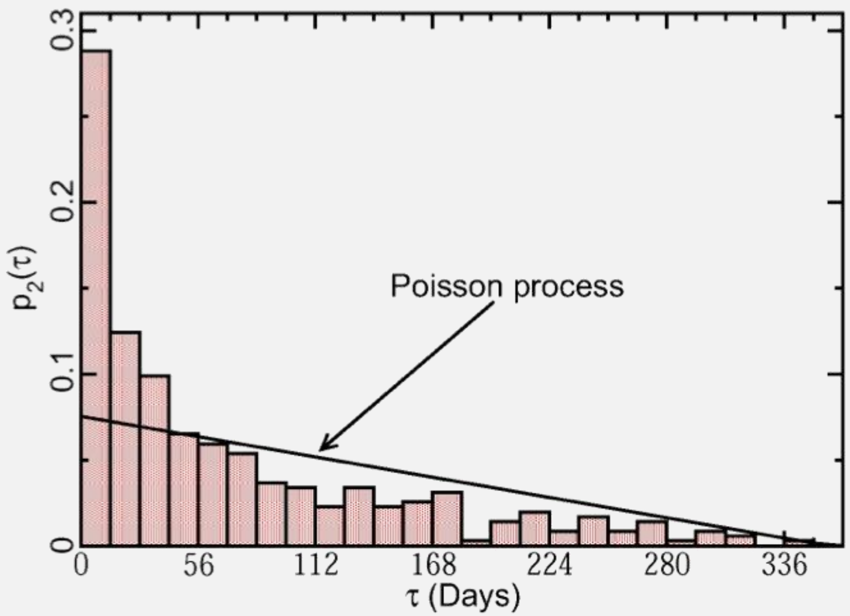
July-September 2001



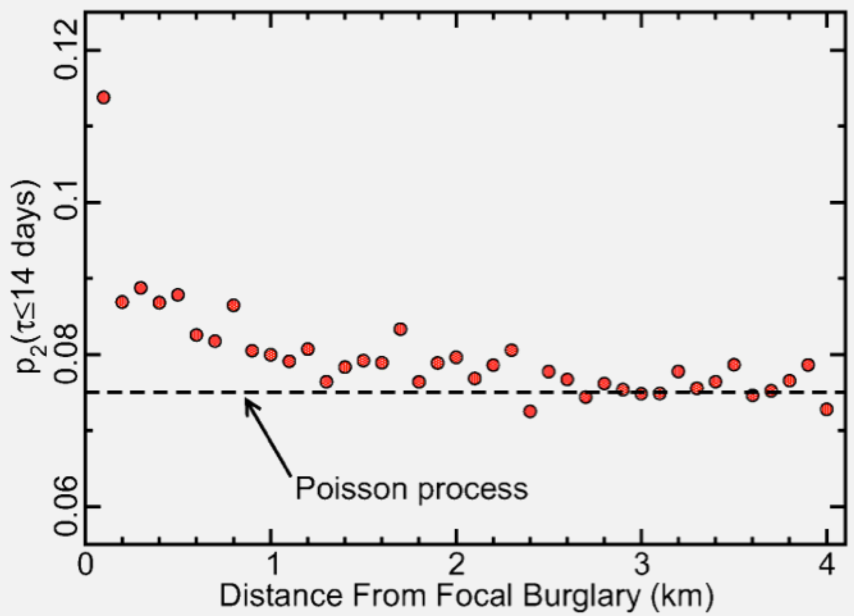
Residential burglaries in Long Beach, CA

First things first: crime exhibits spatial and/or temporal patterning.

Exact repeats

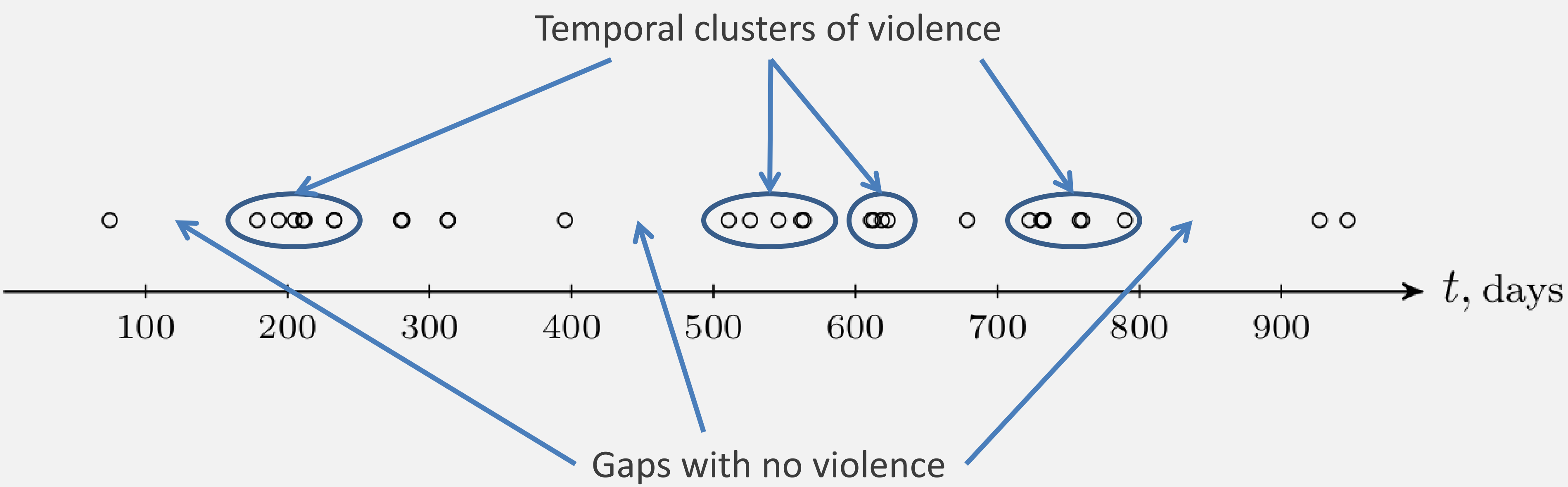


Near repeats



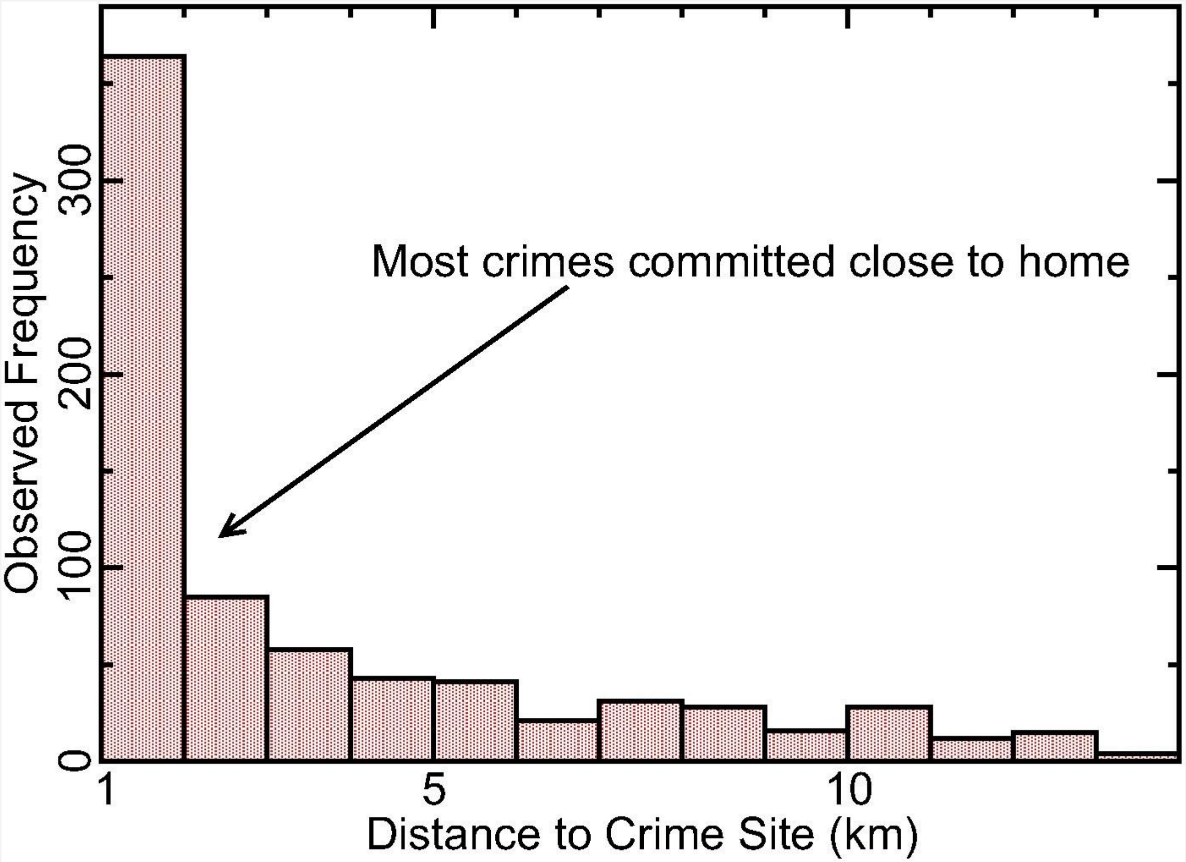
Repeat victimization observations rule out target heterogeneity as the sole cause of patterning.

First things first: crime exhibits spatial and/or temporal patterning.



Violent crimes between Clover and East Lake gangs in Los Angeles, 1999-2002

Secondly, criminals commit crimes close to their home or other "anchor point"



Distance from offender's home to crime for 857 residential burglaries in Long Beach, CA from 2001-2005.

What are the reasons behind this?

- Environmental

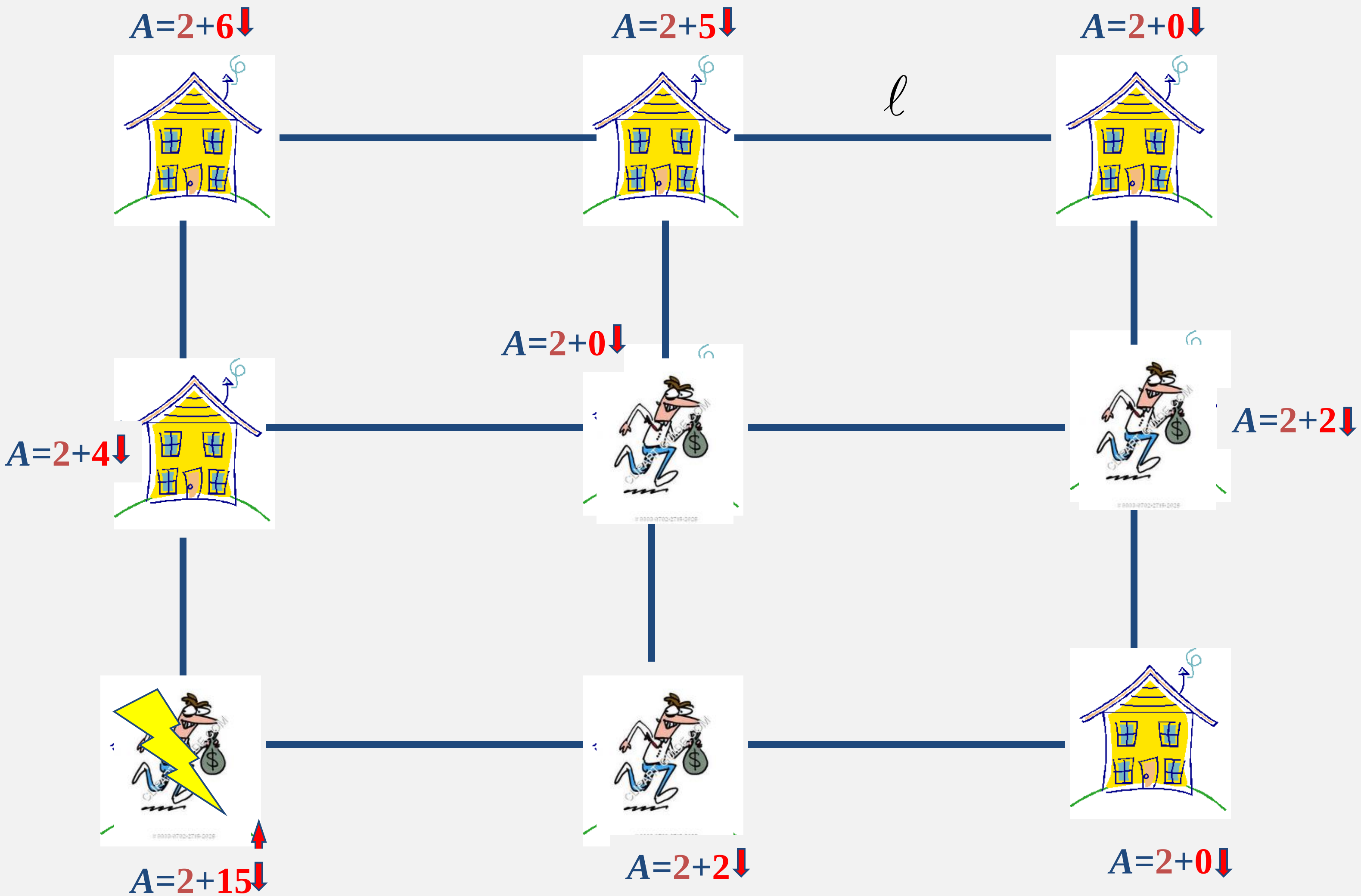
K. Keizer, S. Lindenberg, L. Steg, *Science*, **322** (2008)

- Behavioral
  - “I always go back [to the same places] because, once you been there, you know just about when you been there before and when you can go back. And every time I hit a house, it’s always on the same day [of the week] I done been before cause I know there ain’t nobody there.” (Subject No. 51), Wright and Decker, *Burglars on the Job* (1996)
  - “It's about respect, see. If someone steps up, calls you a [EXPLETIVE] or ‘punk', and your crew [gang] sees that [EXPLETIVE], you better be ready to throw down [fight]. You got to keep your rep [reputation] solid. That's all you got in the `hood'. Your rep. And if someone does come at you, you got to get some back, and get it back hard. If you back down, man, they gonna think you weak.“ (Subject J.T.), Papachristos (2007)
- Routine Activity Theory

The “routine activity” theory of crime

- This theory posits that crimes of opportunity are the most abundant, as opposed to highly planned offences
- Three ingredients needed for a crime:
  - A potential target
  - A potential offender
  - A lack of security
- Crimes can be committed by anyone going about their daily lives, if the opportunity arises.

A grounds-up hotspot model, in cartoon form:

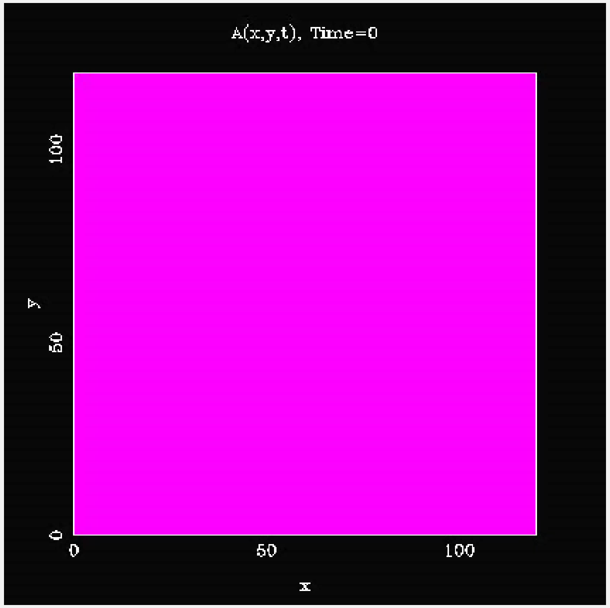


There are many parameters here

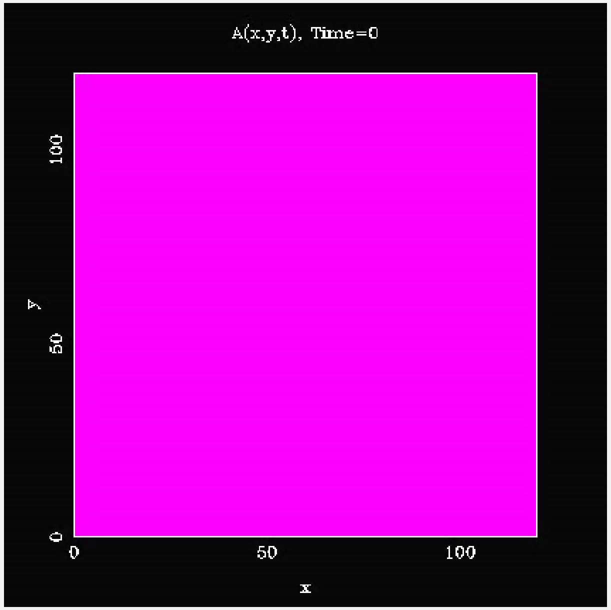
dt	Discrete timestep
ℓ	Lattice spacing
A^0	Intrinsic Attractiveness
ω	Attractiveness decay rate
Θ	Attractiveness increase
η	Spatial spread of A
Γ	Rate of burglar generation

In simulations, we see three types of behavior:

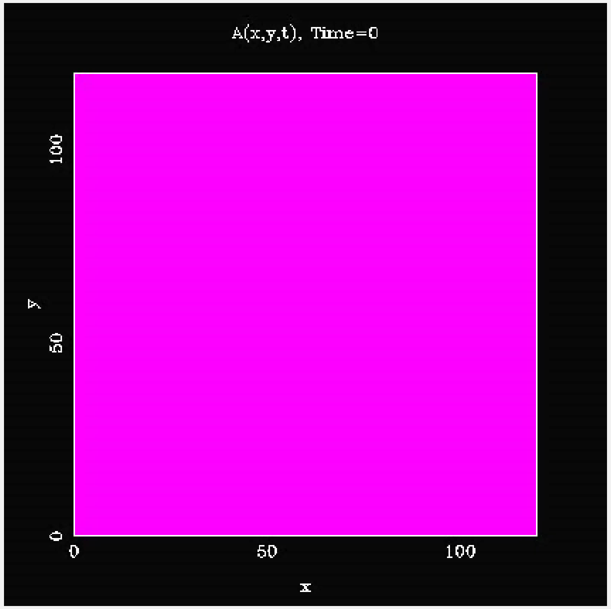
Uniform crime



Moving hotspots



Fixed hotspots



Computer simulations of the cartoon model with different parameters

From a more mathematical perspective

Starting from the discrete agent model, we derive the following continuum PDEs for target attractiveness A and criminal density ρ

Near repeats   Decay to baseline   Exact repeats

$$\frac{\partial A}{\partial t} = \eta \Delta A - (A - A^0) + \rho A$$

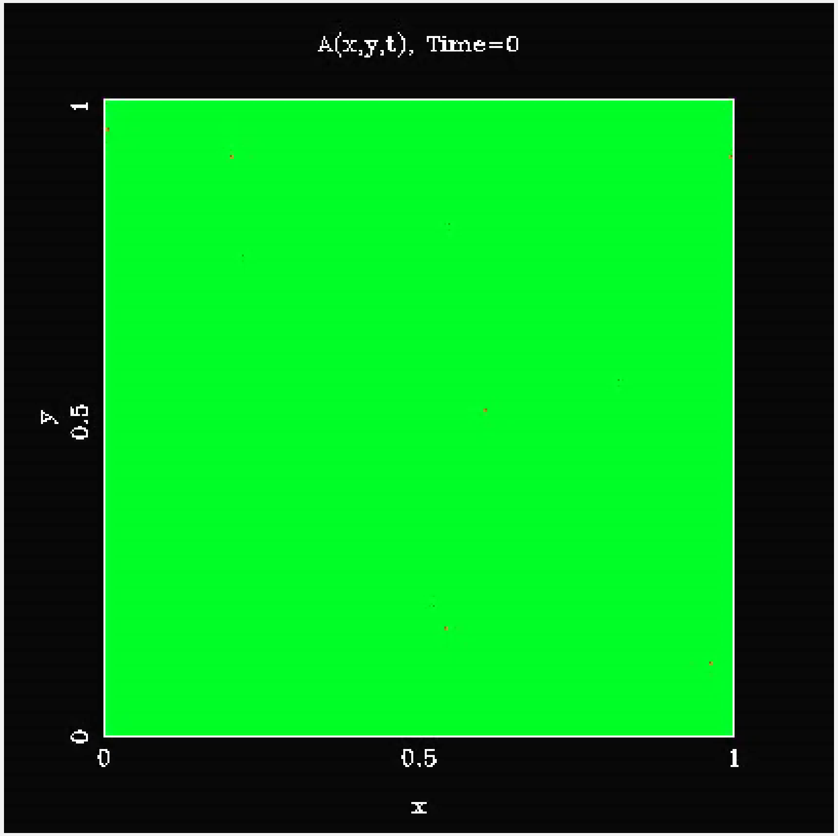
Regeneration

$$\frac{\partial \rho}{\partial t} = \vec{\nabla}\left(\vec{\nabla}\rho - 2\rho\vec{\nabla}\ln A\right) - \rho A + (\bar{A} - A^0)$$

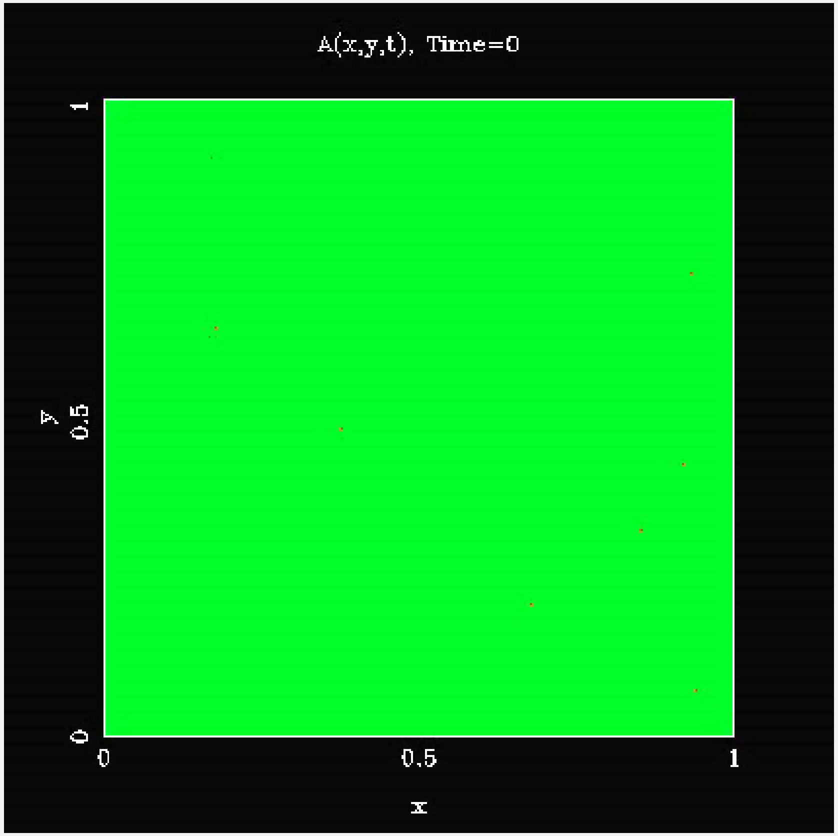
Random motion   Bias toward high A Removal after crimes

In this model, hotspots arise as a bifurcation off the uniform crime state, as parameters vary.

Uniform crime



Crime hotspots



Numerical simulations of the hotspot PDEs with different parameters

Gaining insight: linear stability analysis

- Assume a spatially uniform value for A^0. The homogeneous equilibrium solution is then:

$$\boxed{\bar{A} = A^0 + \bar{B}, \quad \bar{\rho} = \frac{\bar{B}}{A^0 + \bar{B}}}$$

- Perturb this solution via

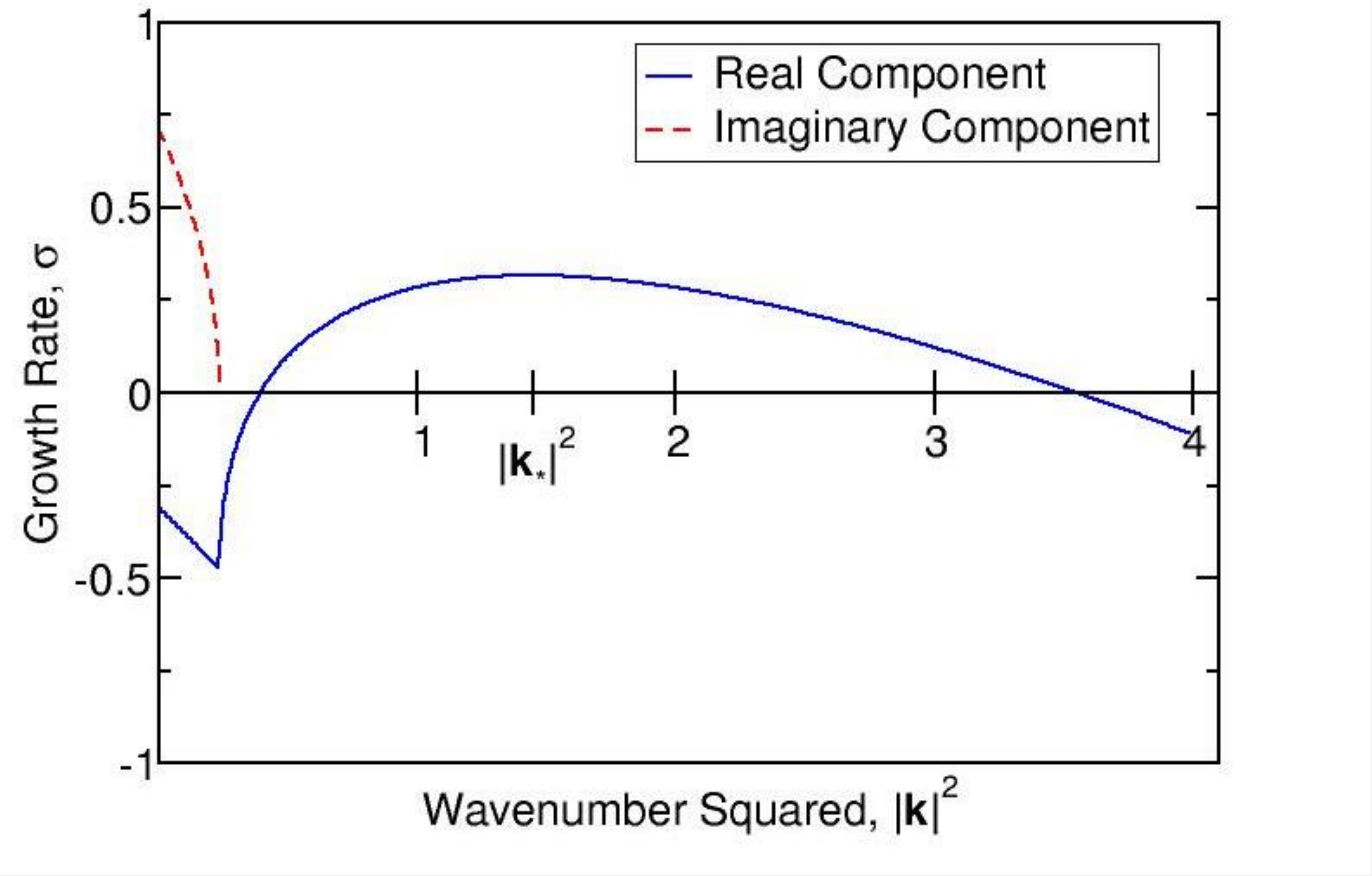
$$\boxed{A = \bar{A} + \delta_a e^{\sigma t} e^{i\mathbf{k}\cdot\mathbf{x}}, \quad \rho = \bar{\rho} + \delta_\rho e^{\sigma t} e^{i\mathbf{k}\cdot\mathbf{x}}}$$

keeping only linear terms in δ_a and δ_ρ

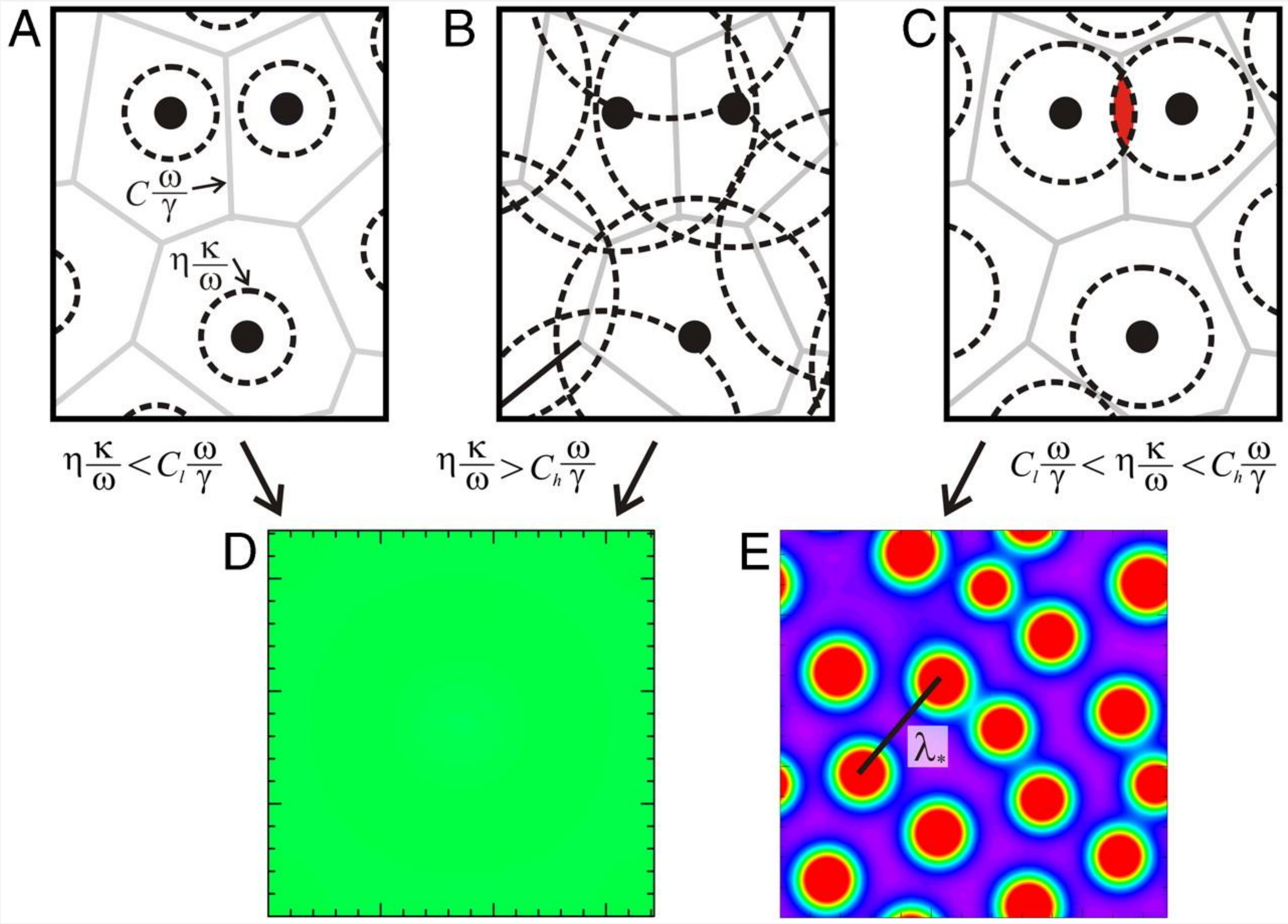
- What is the dispersion relation $\sigma(k)$? Does our perturbation grow or decay?

The hotspot dispersion relation

Linearly unstable if $A^0 < A^0_* \equiv \frac{2}{3}\bar{A} - \frac{1}{3}\eta\bar{A}^2 - \frac{2}{3}\bar{A}\sqrt{\eta\bar{A}}$



A "physical" interpretation



More is found through a weakly nonlinear analysis

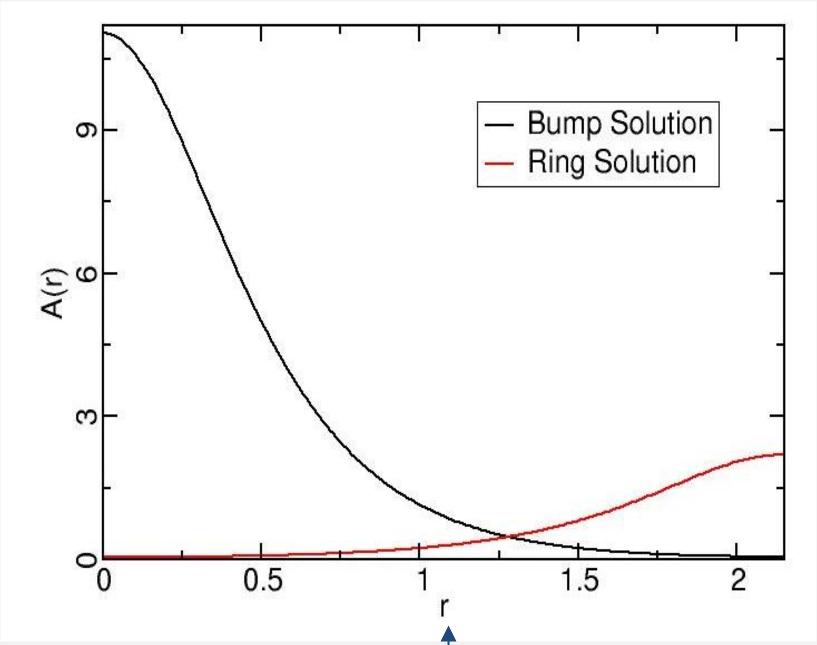
- Assume that we are just slightly unstable, such that

$$\boxed{A^0 = A^0_* - \varepsilon\overline{A}}$$

where ε is a small control parameter that measures how unstable (or stable) the system is

- Define a new, slow timescale $T=\varepsilon t$ and express A and ρ as expansions in ε (that depend on geometry). Solve the resultant differential equations order by order in ε, thus deriving a nonlinear amplitude equation for A.

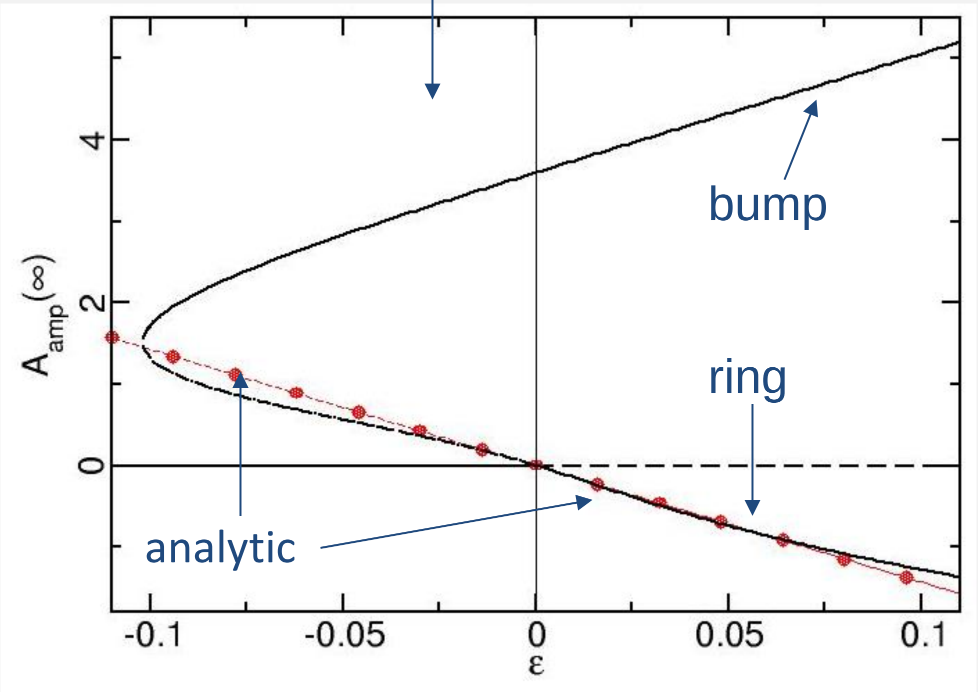
Full nonlinear behavior: 2D, radially symmetric



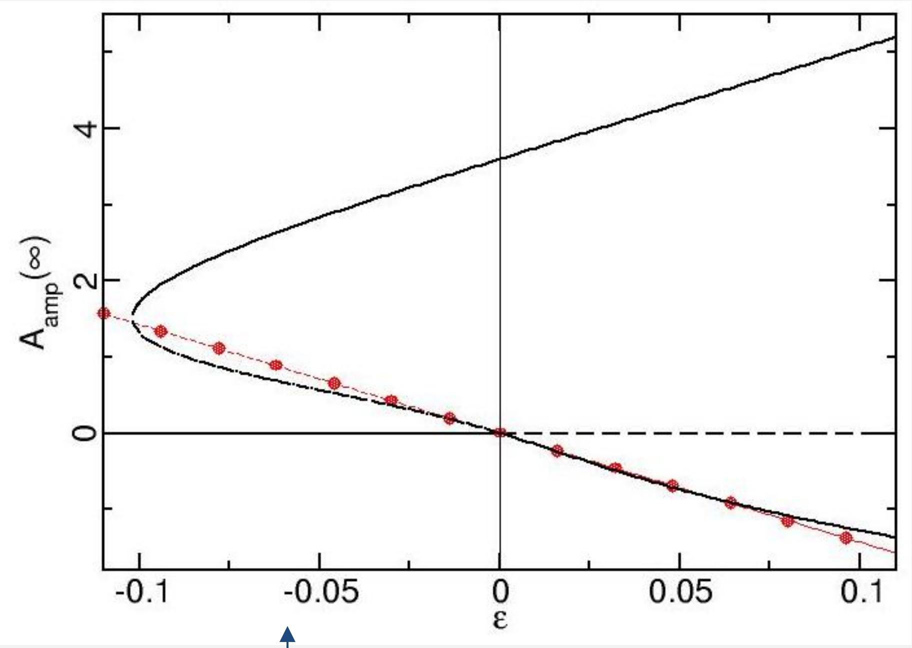
What the steady state solutions look like - bumps and rings

$$\boxed{A(r) = \overline{A} - \varepsilon J_0(k_* r)}$$

Numerical steady state solutions validate theory at small ε. We also see that the bump solution can stably exist over some range of negative ε values, as well as for all positive ε.

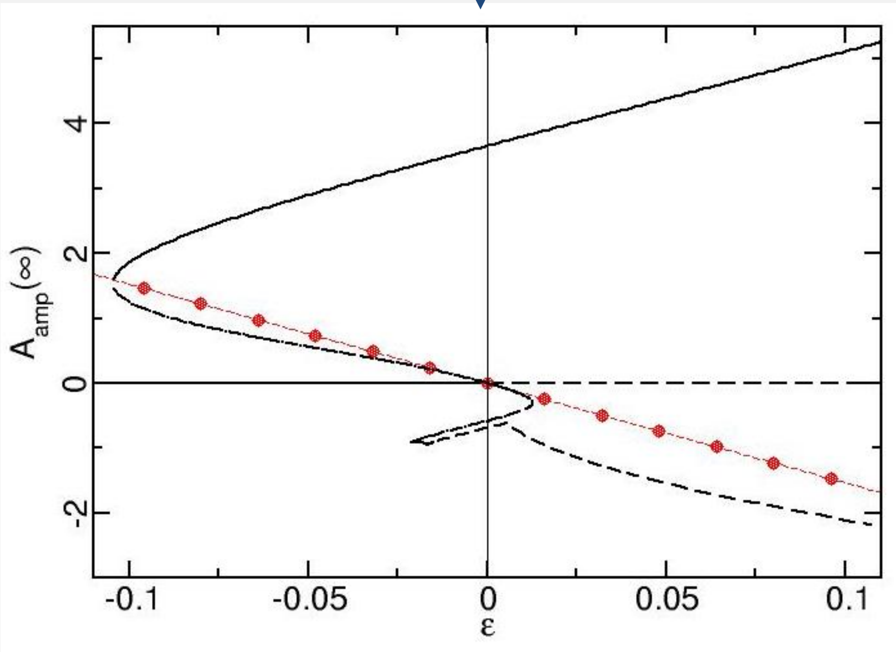


Compared to hexagonal geometry

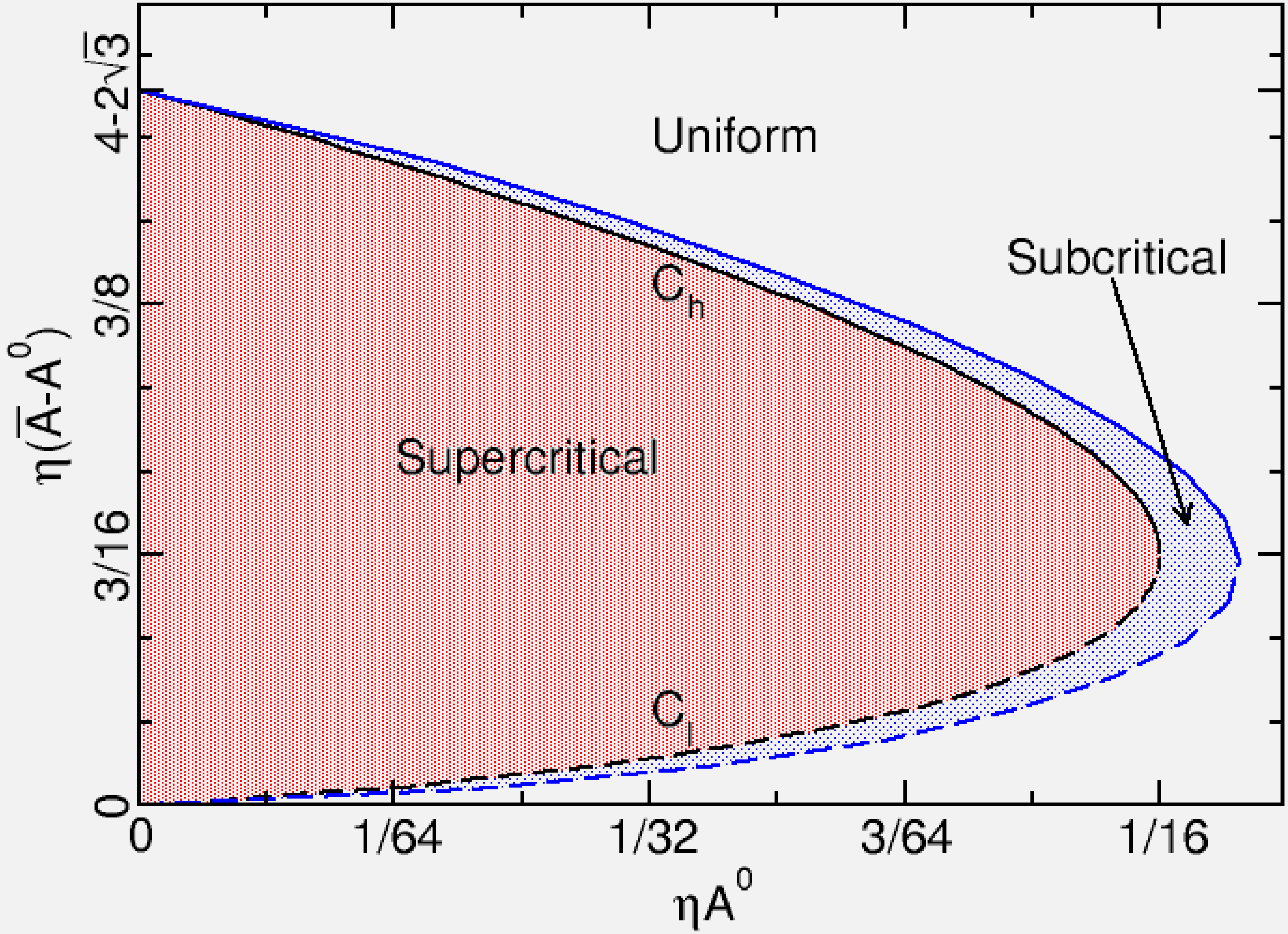


Radially symmetric

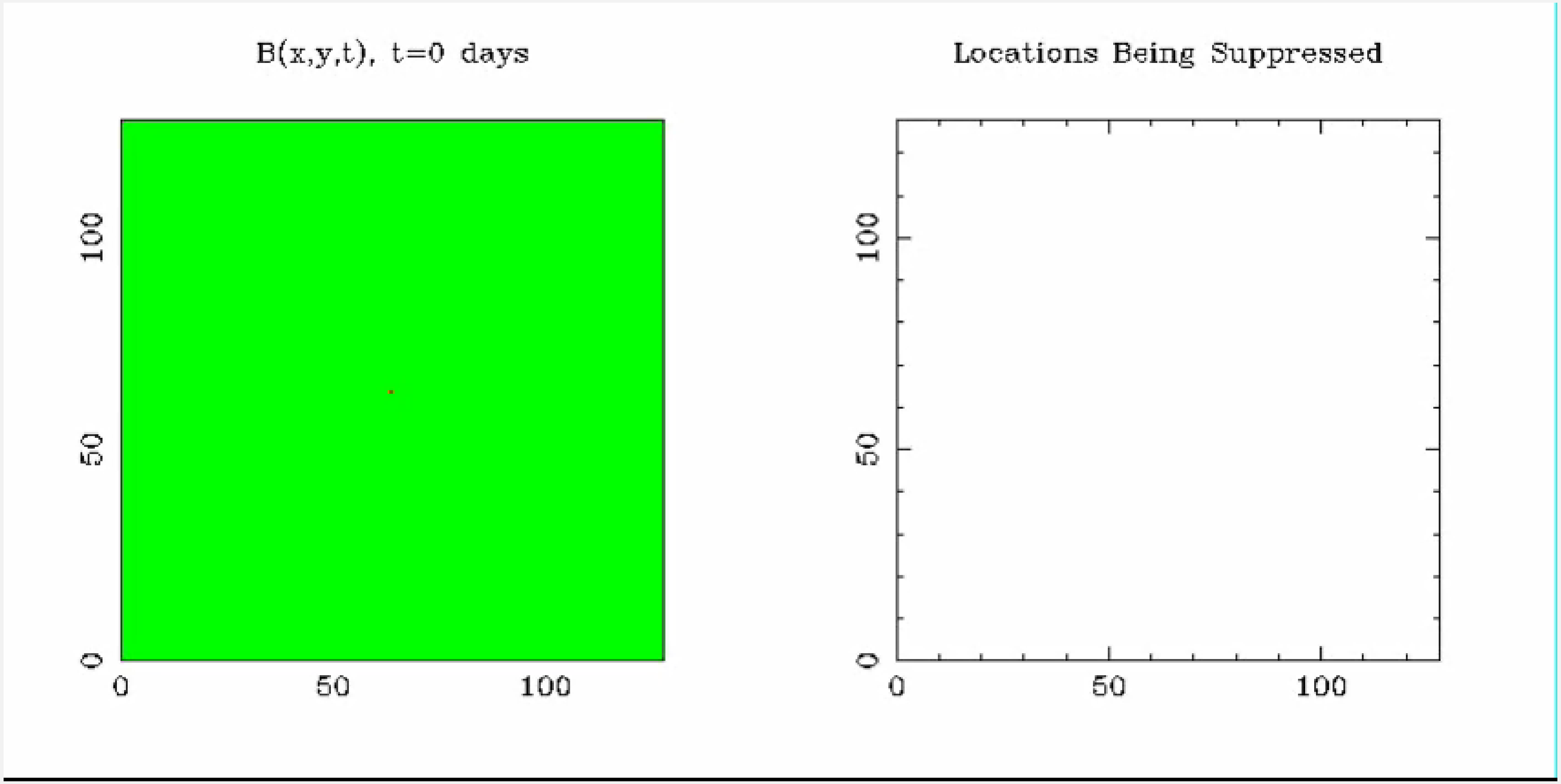
Hexagons
This is the preferred geometry in 2D.
The “ring” solution is not stable here.



A stability diagram summarizes the results

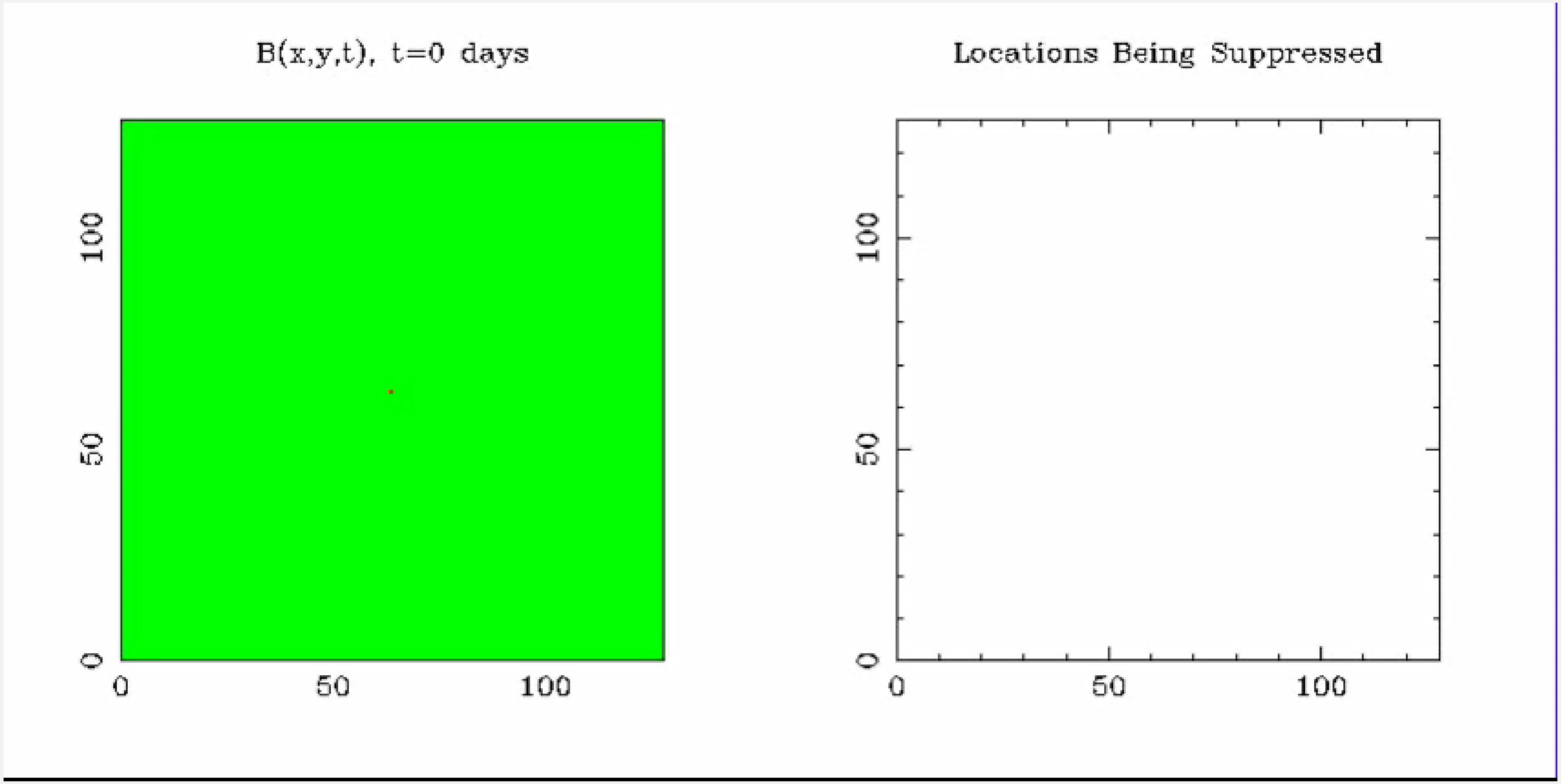


Supercritical and subcritical hotspots respond differently to attempts at suppression.



Supercritical spots respond by displacing

Supercritical and subcritical hotspots respond differently to attempts at suppression.



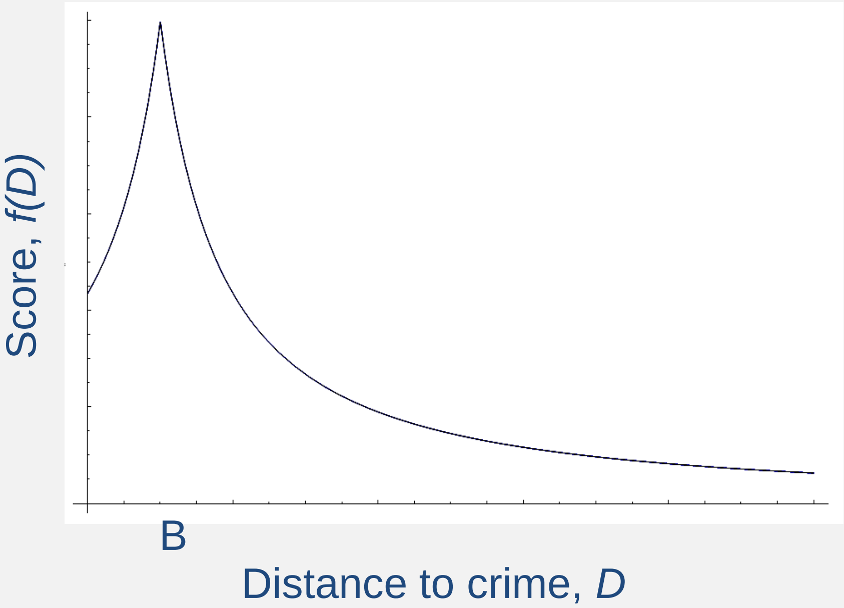
Subcritical spots respond by dissipating

Modeling task 2: Geographic profiling

- Problem: given the crime locations $\mathbf{x}_i$ for a serial offender, predict the location $\mathbf{z}$ of his "anchor point"
- A plausible task, given known distance to crime distributions
- Various algorithms exist to do this. Most widely used is perhaps Rossmo's CGT (Criminal Geographic Targeting) algorithm.
- But, they are unsatisfying…

The CGT algorithm is simple – perhaps too much so

1. Posit a distance to crime "score" function *f(D)*. Parameter *B* is half of mean nearest-neighbor distance for crimes in question

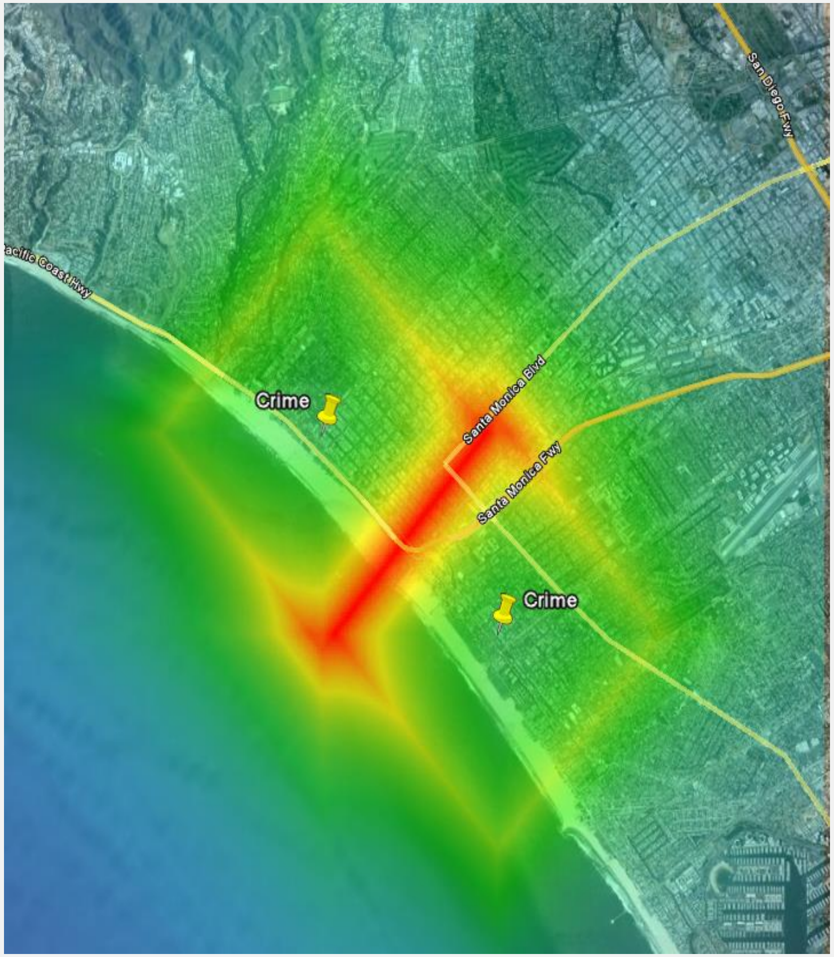


2. For each point **y** on a map, calculate the score *S* at that point via

$$\boxed{S(\mathbf{y}) = \sum_i f\left(\left|\mathbf{y} - \mathbf{x}_i\right|\right)}$$

3. Interpret *S*(**y**) as the likelihood that point **y** is the anchor point for this criminal

This assumes target homogeneity and ignores geographic features

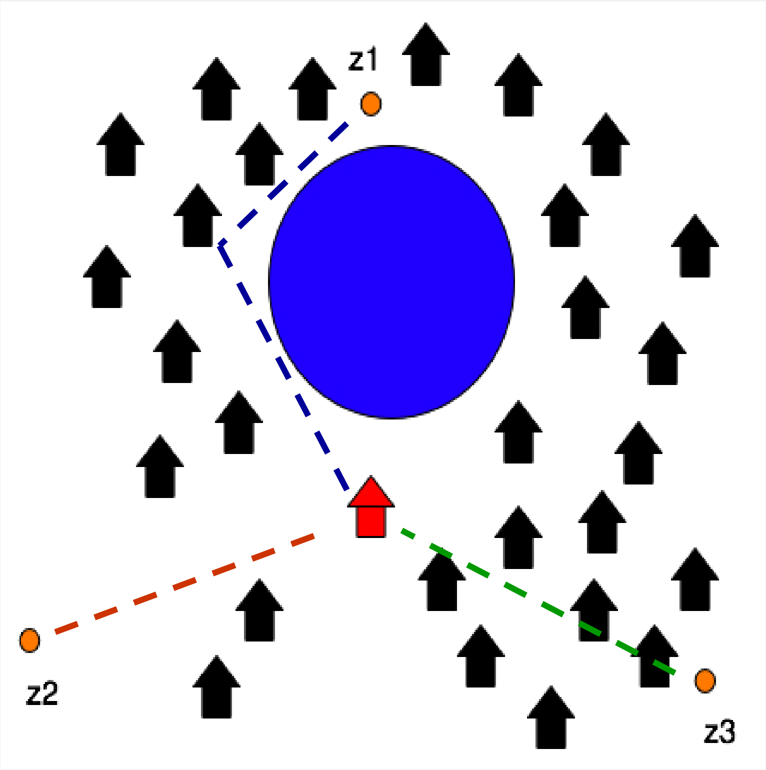


- Many high score spots may be impossible anchor points
- Ignores physical barriers between crime locations and potential anchor points such as lakes, mountains, highways, etc…
- No accounting for variability in locations of other crime targets

Our approach: model basic criminal behavior and **recover** distance decay

- From routine activity theory – criminals will simply be people moving about their environment in quasi-realistic ways, starting from their anchor point
- We assume Brownian motion, with possible drift. Other options are readily implemented.
- Criminals "interact" with the local crime rate $A(\mathbf{x})$, which determines whether they commit a crime or not at point $\mathbf{x}$, at which point they head back home
- This is just the criminal portion of the agent-based model proposed earlier!

This method includes heterogeneity of targets and environment naturally



Of these three possible home locations:

- - - : low prob. - path to the crime site is long and full of other targets

- - - : medium prob. - path is short but still full of other targets

- - - : high prob. - path is short with few targets

Finally, the mathematics

- In the continuum sense, the probability $\rho(t, \mathbf{x} \mid \mathbf{z})$ that the criminal is located at point **x** at time *t* is given by the solution of the Fokker-Planck equation:

$$\boxed{\frac{d\rho}{dt} = \nabla \cdot \left(D(\mathbf{x}) \nabla \rho - \vec{\mu}(\mathbf{x}) \rho \right) - A(\mathbf{x}) \rho, \quad \rho(0, \mathbf{x}) = \delta(\mathbf{x} - \mathbf{z})}$$

- Hence, the probability $P(\mathbf{x} \mid \mathbf{z})$ that a criminal with anchor point **z** commits a crime at **x** is

$$\boxed{P(\mathbf{x} \mid \mathbf{z}) = A(\mathbf{x}) \rho(\mathbf{x} \mid \mathbf{z}), \quad \rho(\mathbf{x} \mid \mathbf{z}) = \int_0^\infty \rho(t, \mathbf{x} \mid \mathbf{z}) \, dt}$$

Could be implemented at this point

1. Divide your map into a number of cells, each representing a possible anchor point **z**
2. For each potential anchor point, find $P(\mathbf{x}_i \mid \mathbf{z})$ for each of the *N* crimes in the series
3. Assuming *event independence*, the probability that the cell at **z** is the anchor point is

$$\boxed{P(\mathbf{z}) \propto Q(\mathbf{z}) \prod_{i=1}^{N} P\left(\mathbf{x}_i \mid \mathbf{z}\right)}$$

where $Q(\mathbf{z})$ is a prior distribution of anchor points.

But, this method is wasteful and slow, especially if the number of potential anchor points is high…

The backward equation is, therefore, more useful

- $\rho(\mathbf{x} \mid \mathbf{z}) = f(\mathbf{z} \mid \mathbf{x})$ satisfies the adjoint equation:

$$\boxed{-\nabla \cdot \left(D(\mathbf{z}) \nabla f\right) - \vec{\mu}(\mathbf{z}) \cdot \nabla f + A(\mathbf{z}) f = \delta(\mathbf{z} - \mathbf{x})}$$

  where the derivatives are now in **z**

- Essentially, we start the criminals at the crime site and "follow" them home
- We only need to calculate $f(\mathbf{z} \mid \mathbf{x})$ for each crime site, then use

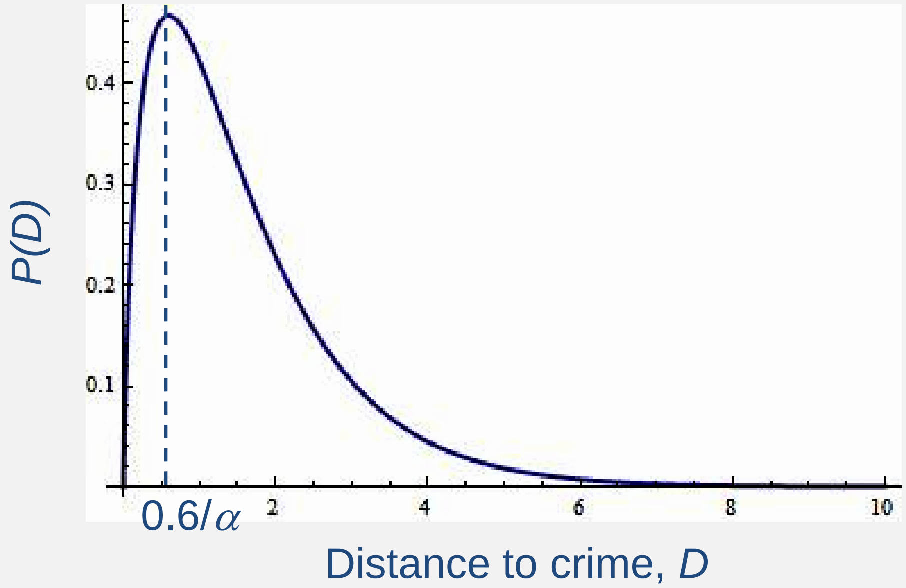
$$\boxed{P(\mathbf{z}) \propto Q(\mathbf{z}) \prod_{i=1}^{N} f\left(\mathbf{z} \mid \mathbf{x}_i\right)}$$

Simplest case: no drift, homogeneous parameters

- Let $\alpha^2 = A / D$. Then

$$\boxed{P(\mathbf{x} \mid \mathbf{z}) = \frac{\alpha^2}{2\pi} K_0(\alpha \mid \mathbf{x} - \mathbf{z} \mid)}$$

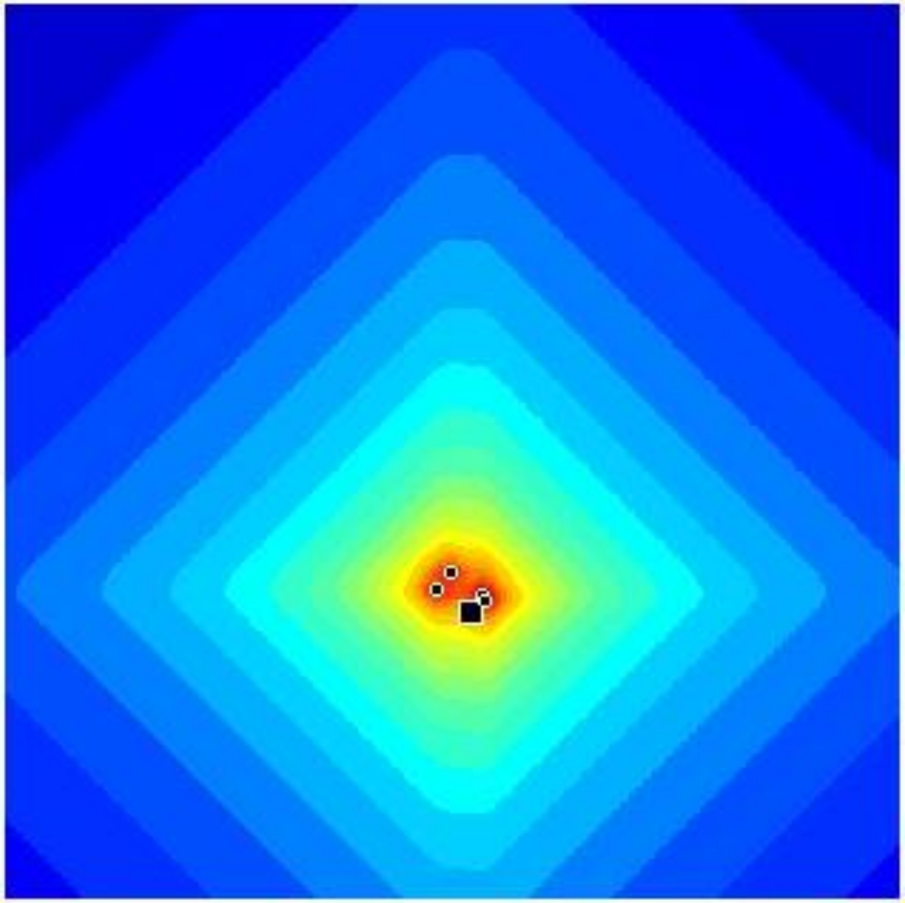
- Distance decay is automatically recovered!



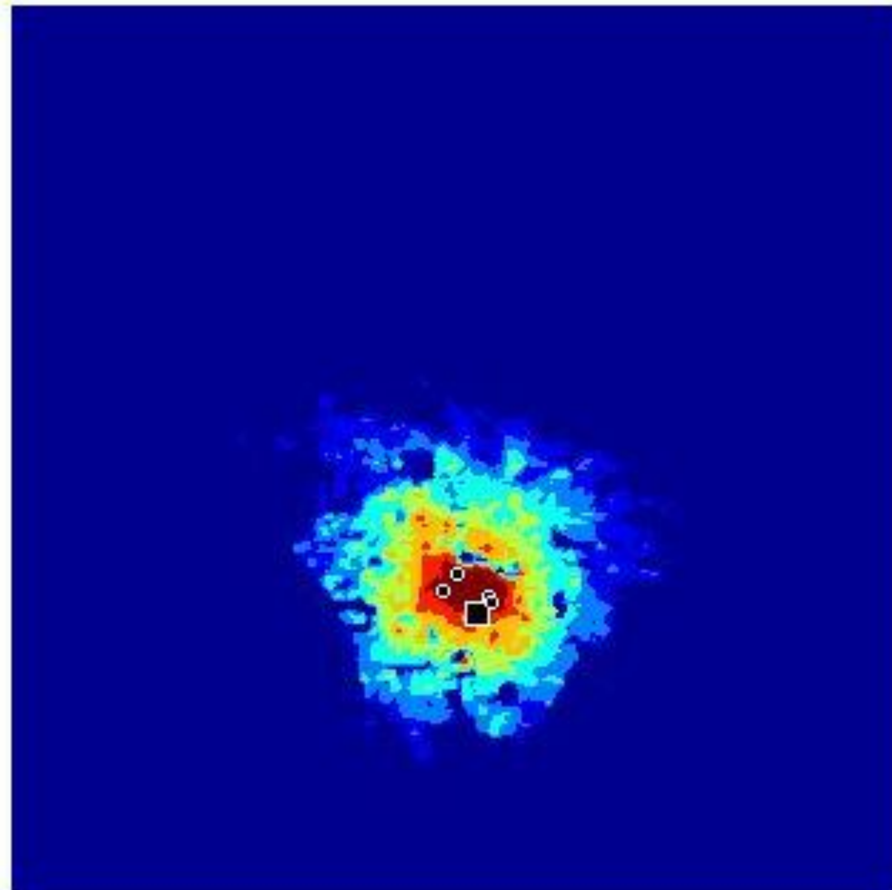
Test case: 244, N>2 residential burglary series from Los Angeles

- We assume no drift, and that D is homogeneous
- We model $A(\mathbf{x})=A_b\, H(\mathbf{x})$, where $H(\mathbf{x})$ is the distribution of residences as taken from the 2000 US Census. $Q(\mathbf{x})$ is the population distribution, also from the Census.
- For each series, we use the other 243 to fit the one effective parameter α using Maximum Likelihood Estimation, then Geoprofile the series in question.

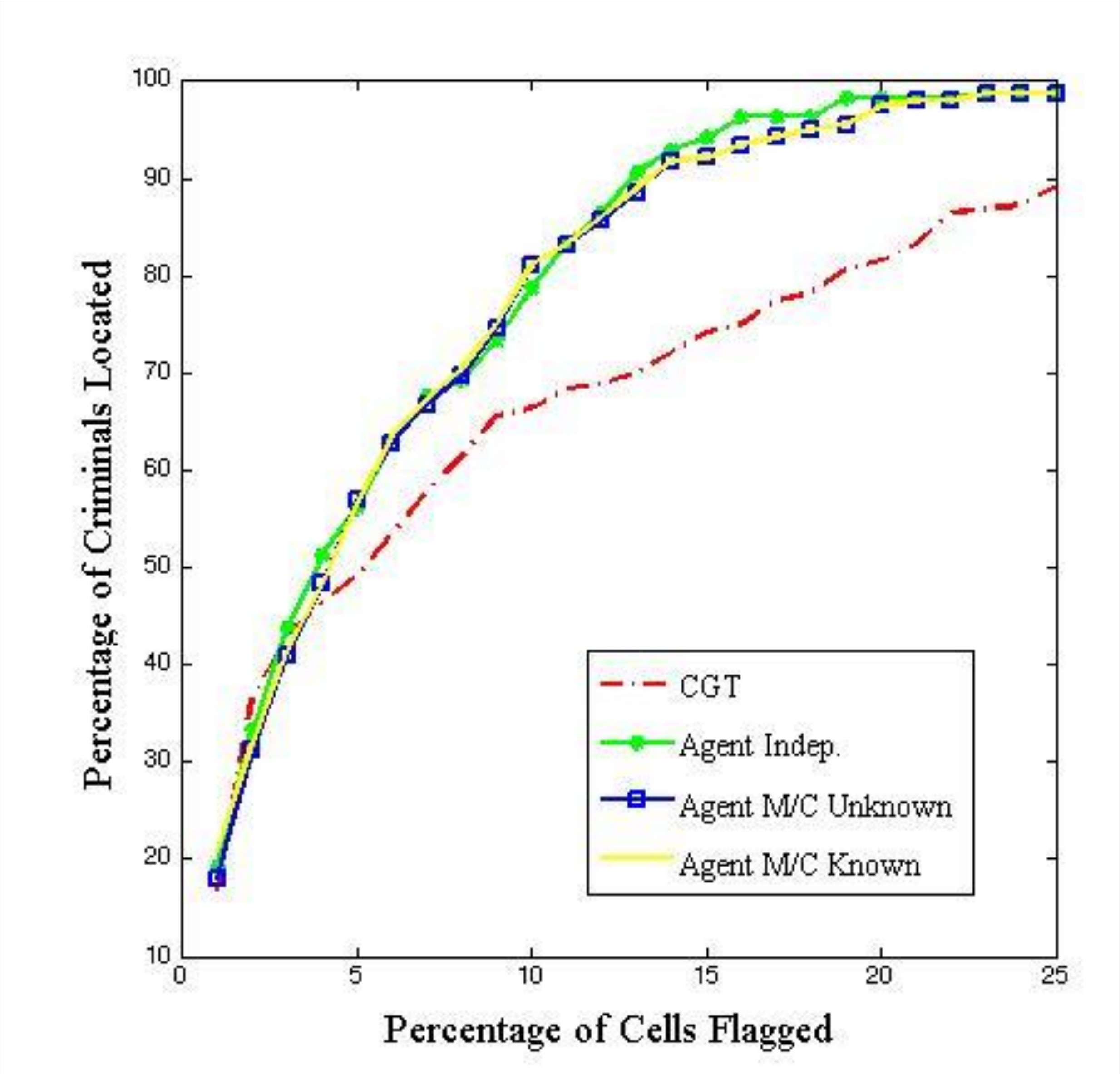
Example Output: CGT Algorithm



Example Output: Our Algorithm



Comparing algorithms via “hit percent”



Conclusions

- Agent-based models seem capable of providing insight into several facets of criminal activity
  - Explaining crime hotspot formation
  - Explaining hotspot displacement versus dissipation
  - Explaining distance decay from home to target
- That said, the continuum versions of these models are often more useful for analysis and computation

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