Synchronization and Swarming: Clocks and Flocks
Andrew J. Bernoff, Harvey Mudd College

Thanks to Chad Topaz, Macalester College
Abstract:

The flocking of starlings, contractions of heart muscle, and fads in culturel and fashion are examples of systems where social and/or biological interactions promote the emergence of coherence from disorder. Temporally, coupling of oscillators leads to the synchronization observed in the flashing of fireflies or the trending of hashtags in twitter. Spatially, coupling leads to biological aggregation such as the swarming of locusts or the schooling of fish. In this talk we will give an overview of some of the ways in which social and biological coupling facilitate collective behavior and describe some of the characteristic of the phenomena they manifest.

Useful Links:

- [Birds Do It. Bees Do It](http://www.news.ucsb.edu/2014/013963/birds-do-it-bees-do-it) This is a news story that includes the human flocking experiments.

- [Synchronization and Swarming of Clocks and Flocks](http://online.kitp.ucsb.edu/online/bioactert-c14/bernoff/) Video of a version of this talk presented at the Kavli Institute of Theoretical Physics on February 15, 2014.

- [A Primer of Swarm Equilibria](http://beta.mbi.ohio-state.edu/video/player/?id=143&title=A+Primer+of+Swarm+Equilibria) Video of a talk presented at the Mathematical Biological Institute at Ohio State University on March 15, 2011.
Synchronization  (noun)

The tendency of periodic systems to align in time.

• Sleep/wake cycles
• Flowering of plants /Estrus in animals
• Clapping (sometimes !!)
• Dancing
• Contraction of muscle in heart cells
• Lasers
• Microwave Ovens
Synchronization of metronomes

Lancaster University, Dept. of Physics
Coupling (noun)

The influence of one object on another; feedback.

- Sight, sound, smell, vibrations
- Pheromones
- The internet
- Vibrations
- Crickets chirping, fireflies flashing
Coupled oscillators

11 x 11 Grid of Oscillators
Strong local coupling
Weak global coupling

32 Metronomes

Synchronization of thirty two metronomes

2012年09月14日，池口研究室前廊下にて撮影
Filmed at Ikeguchi Laboratory, on September 14, 2012.
Modeling Synchronization

\[
\frac{d\theta_i}{dt} = \omega_i + \sum_{j=1}^{N} K_{ij} \sin(\theta_i - \theta_j)
\]

The Kuramoto Model:

\[\theta_i = \text{Phase of } i^{th} \text{ oscillator.}\]
\[\omega_i = \text{Frequency of } i^{th} \text{ oscillator.}\]
\[K_{ij} = \text{Coupling of } i^{th} \text{ to } j^{th} \text{ oscillator.}\]

Note coupling may depend on distance or topology.
Meme  (noun)

An idea or behavior that spreads through cultural coupling.

• Fashion
• Fads
• #hashtags
• Religion
• Kale
• Sriracha
• Bitcoins
7. A mathematical model of Bieber Fever: The most infectious disease of our time?

Valerie Tweedle$^1$ and Robert J. Smith$^2$

$^1$Department of Biology, The University of Ottawa, 585 King Edward Ave, Ottawa ON K1N 6NS, Canada; $^2$Department of Mathematics and Faculty of Medicine, The University of Ottawa, 585 King Edward Ave, Ottawa ON K1N 6NS, Canada
Most popular names of baby girls in each state by year
Modeling Memes

**S-I-R Model**

\[
\frac{dS}{dt} = -\frac{\beta IS}{N} \\
\frac{dI}{dt} = \frac{\beta IS}{N} - \gamma I \\
\frac{dR}{dt} = \gamma I
\]

\( S = \text{Susceptible} \)  
\( I = \text{Infected} \)  
\( R = \text{Recovered (or removed)} \)

Note coupling may depend on distance or topology.

http://en.wikipedia.org/wiki/Compartmental_models_in_epidemiology
What have we learned so far . . .

- **Coupling** can lead to **synchronization**.
- The **coupling** of human behavior can lead to **memes**.
- Humans are definitely **coupled** - we seem programmed to imitate each other.

What does this mean for animal behavior?

Want more? See Steve Strogatz’s TED talk & book!!
Coupling of behavior leads to swarming
Swarm (noun)

The grouping (aggregation) of coupled objects, often animals.

- Bird flocks
- Animal herds
- Schools of fish
- Insect swarms (bees, ants, locusts)
Biological groups move in a coordinated manner.
Groups can propagate without a leader.
Groups may have sharp edges, nearly constant density

Aggregation (noun)

The grouping of coupled objects, often animals.

- Bird flocks
- Animal herds
- Schools of fish
- Insect swarms (bees, ants, locusts)
- Robotic swarms
1. Individuals are **attracted** toward neighbors.

2. Individuals are **repelled** if they get too close.
Human Swarming I

**WALK** slowly toward (what you perceive to be) the center of the group.

**SLOW DOWN** if you are within two feet of another person.

**STOP** if you are within one foot of another person.
Modeling Aggregations

\[ \dot{x}_i = \left( \sum_{j=1}^{N} \frac{dp}{dr} \left( \left| \vec{r}_{ij} \right| \right) \frac{\vec{r}_{ij}}{\left| \vec{r}_{ij} \right|} \right) \]

pairwise additive interactions

Interorganism Potential \( p(r) \)

\[ p(r) = -F L e^{-r/L} + e^{-r} \]

long-range attraction  
short-range repulsion

Kinematic Model: Velocity = \( \sum \) Social Force
Mill

A *rotating circular swarm*.

- Ants
- Fish
- Occasionally people
Individual behavior can lead to large scale pattern formation

Fish school
(Parrish & Keshet, 1999)

Ant mill
(Schnierla, 1971)

Dynamic Model with
Momentum, Attraction & Repulsion
D'Orsogna, Chuang, Bertozzi & Chayes

Dynamic Model:
Acceleration = Propulsion - Drag Forces + Social Forces
Human Swarming II

**WALK** at a slow, constant speed.

**WALK** toward the person or people you see in front of you.

**TURN RIGHT** if you are going to collide.
Humans can spontaneously exhibit mill behavior also
Frontiers

The limits of knowledge - where the action is.
Frontier #1: Efficient computation of swarms

N = 1 thousand

N = 1 million

GPU Computing

Difficulty: Pairwise interactions means calculation scales like \( N^2 \)
Frontier #2: Quantitative lab/field measurements

Inferring individual rules from collective behavior
Lukeman, Li & Edelstein-Keshet
(PNAS 2010)

What is a minimal model for duck movement?
Frontier #2: Quantitative lab/field measurements

Motion tracking of pea aphids

XMAC lab - Chad Topaz, Macalester College
What is a minimal model for pea aphid movement?
Frontier #3: Modeling human behavior

How well can we model human interactions?
Agent Based Modeling: A Top Ten List
#10: Understand your goal.

# 9: Have a metric in mind.

# 8: Deal with the curse of dimensionality

# 7: Experimentalists are your friends

# 6: Think about the geometry of coupling.
# 5: Think laterally and borrow mercilessly.

# 4: Be curious and observant

# 3: Particles or Continuum Model?

# 2: Random vs. Deterministic

# 1: Parsimony !!