Synchronization and Swarming: Clocks and Flocks

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Abstract:

The flocking of starlings, contractions of heart muscle, and fads in culturel and fashion are examples of systems where social and/or biological interactions promote the emergence of coherence from disorder. Temporally, coupling of oscillators leads to the synchronization observed in the flashing of fireflies or the trending of hashtags in twitter. Spatially, coupling leads to biological aggregation such as the swarming of locusts or the schooling of fish. In this talk we will give an overview of some of the ways in which social and biological coupling facilitate collective behavior and describe some of the characteristic of the phenomena they manifest.

Useful Links:

- Birds Do It. Bees Do It [http://www.news.ucsb.edu/2014/013963/birds-do-it-bees-do-it]
  This is a news story that includes the human flocking experiments.
- Synchronization and Swarming of Clocks and Flocks [http://online.kitp.ucsb.edu/online/bioactert-c14/bernoff/]
  Video of a version of this talk presented at the Kavli Institute of Theoretical Physics on February 15, 2014.
- A Primer of Swarm Equilibria [http://beta.mbi.ohio-state.edu/video/player/?id=143&title=A+Primer+of+Swarm+Equilibria]
  Video of a talk presented at the Mathematical Biological Institute at Ohio State University on March 15, 2011.

Synchronization *(noun)*

The tendency of periodic systems to align in time.

- Sleep/wake cycles
- Flowering of plants /Estrus in animals
- Clapping (sometimes !!)
- Dancing
- Contraction of muscle in heart cells
- Lasers
- Microwave Ovens

Synchronization of metronomes

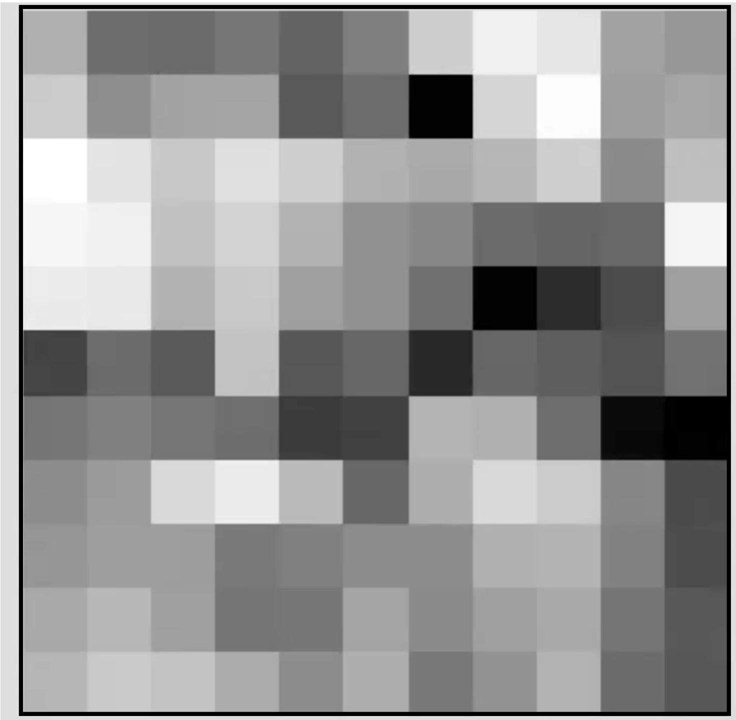
Lancaster University, Dept. of Physics

Coupling *(noun)*

The influence of one object on another; feedback.

- Sight, sound, smell, vibrations
- Pheromones
- The internet
- Vibrations
- Crickets chirping, fireflies flashing

Coupled oscillators



11 x 11 Grid
of Oscillators

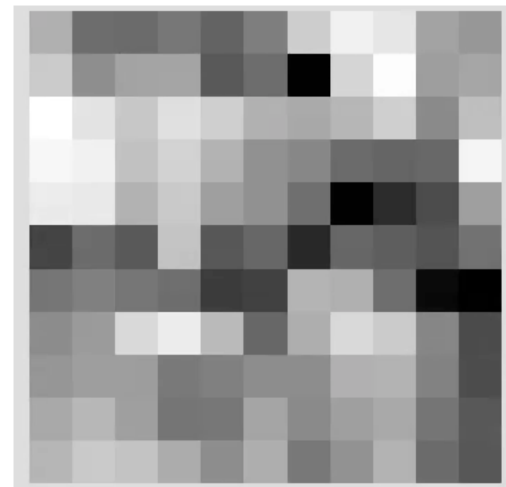
Strong local coupling
Weak global coupling

32 Metronomes

Modeling Synchronization

$$\frac{d\theta_i}{dt} = \omega_i + \sum_{j=1}^{N} K_{ij} \sin(\theta_i - \theta_j)$$

The Kuramoto Model:

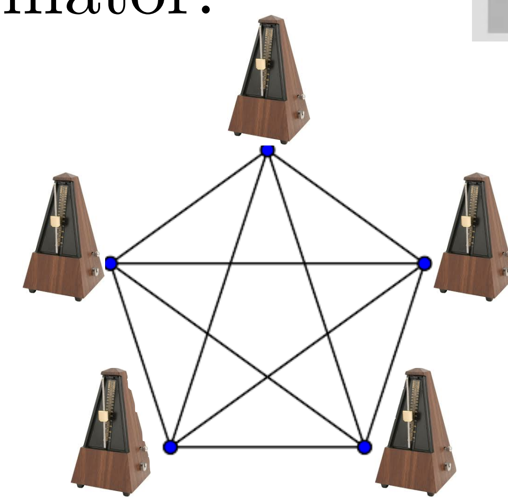


$\theta_i =$ Phase of i^{th} oscillator.

$\omega_i =$ Frequency of i^{th} oscillator.

$K_{ij} =$ Coupling of i^{th} to j^{th} oscillator.

Note coupling may depend on distance or topology.

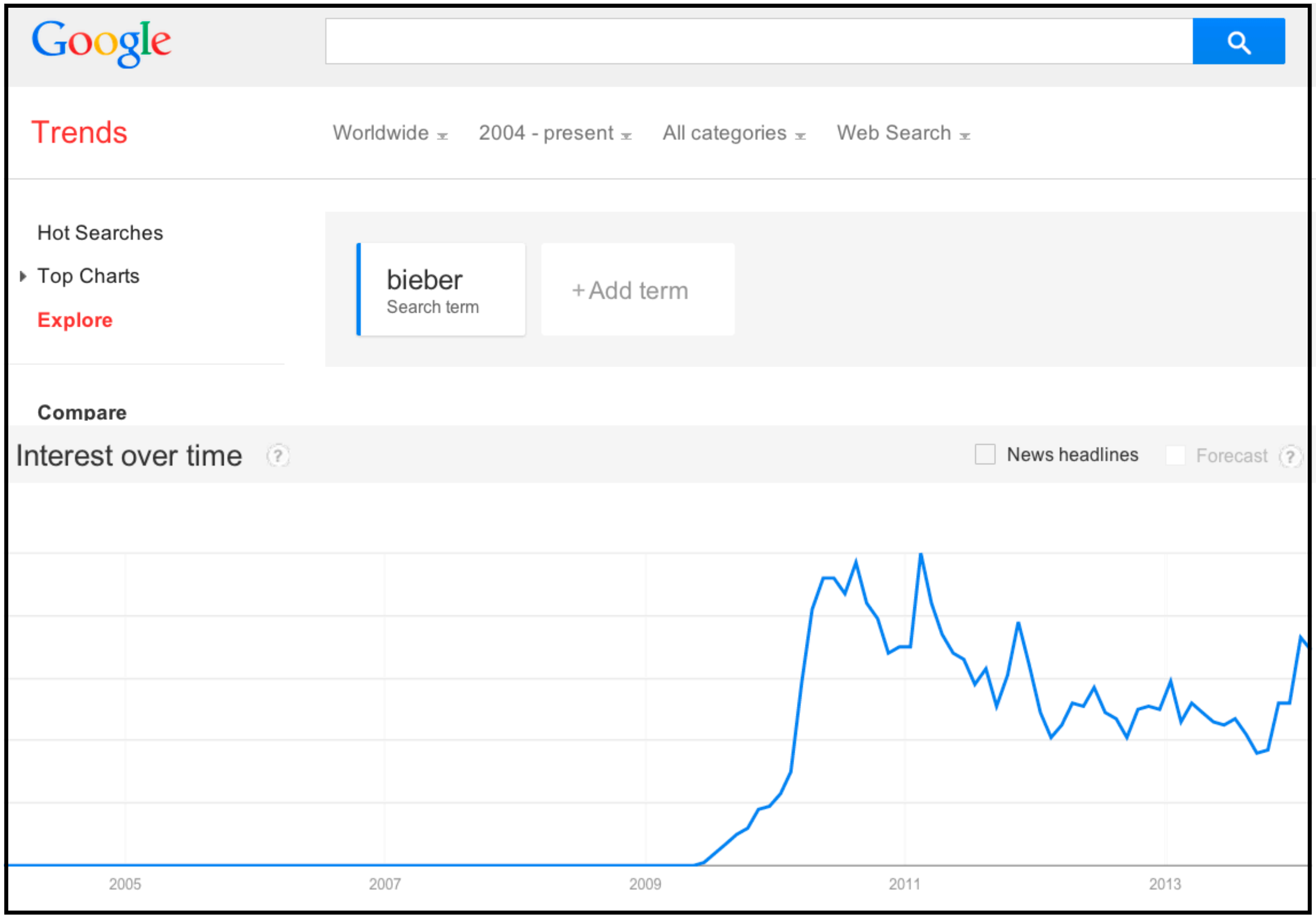


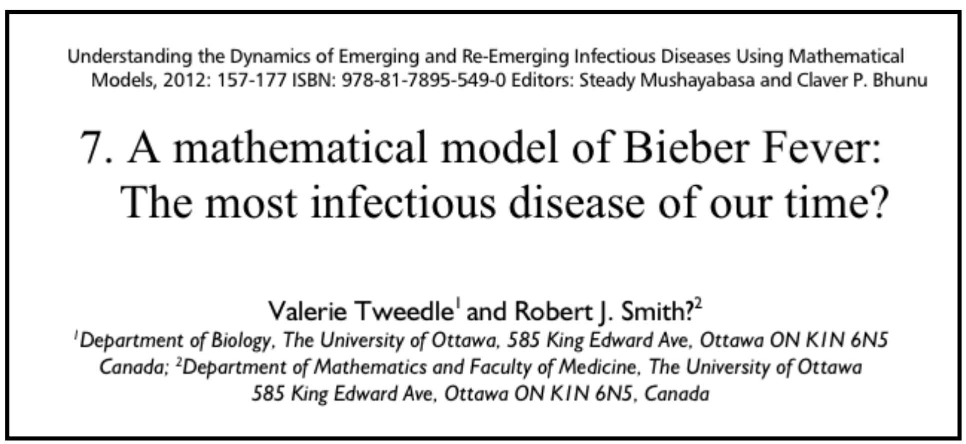
Meme *(noun)*

An idea or behavior that spreads through cultural coupling.

- Fashion
- Fads
- #hashtags
- Religion
- Kale
- Sriracha
- Bitcoins

Bieber Fever (Meme)





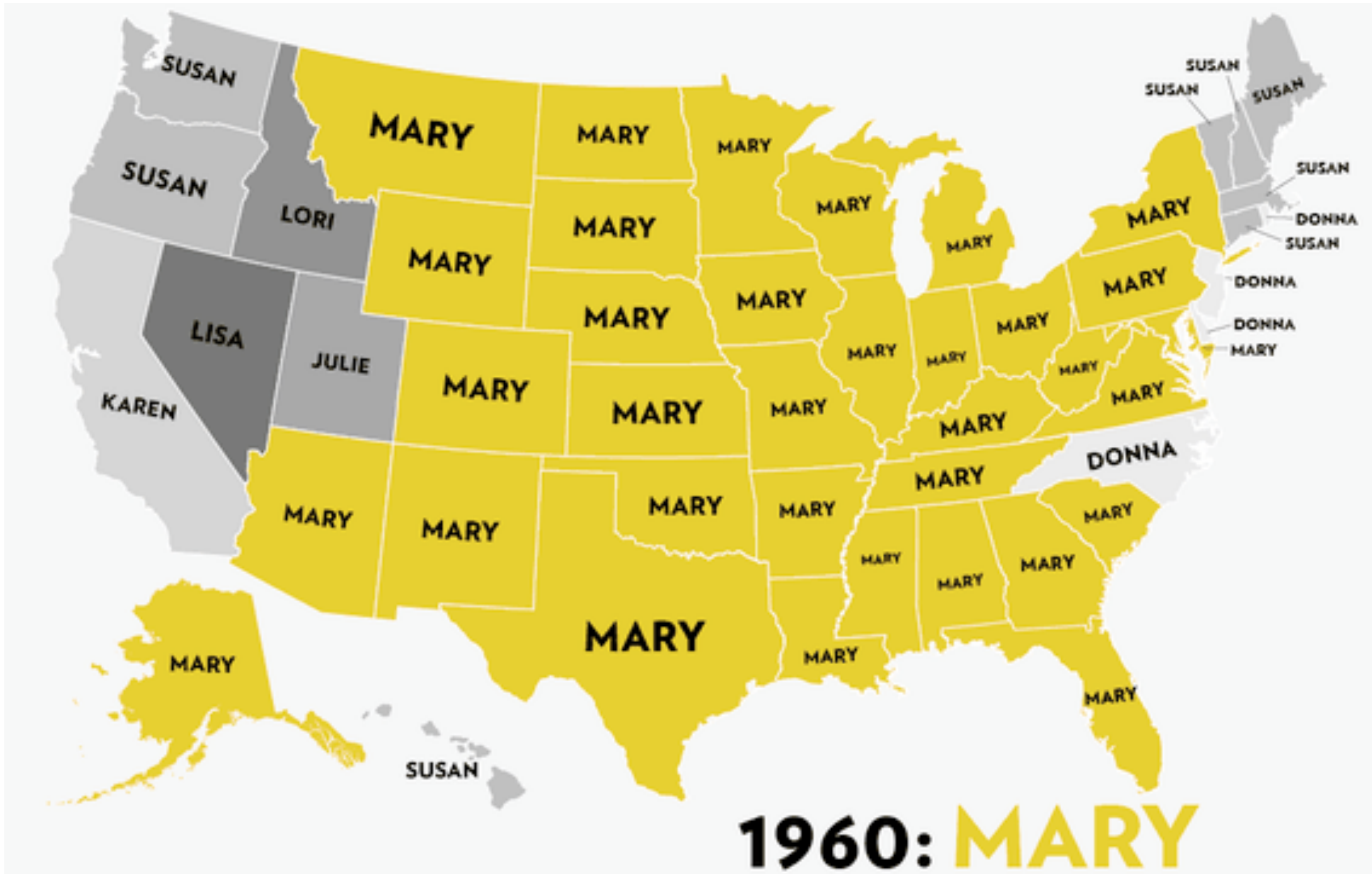
Understanding the Dynamics of Emerging and Re-Emerging Infectious Diseases Using Mathematical Models, 2012: 157-177 ISBN: 978-81-7895-549-0 Editors: Steady Mushayabasa and Claver P. Bhunu

7. A mathematical model of Bieber Fever: The most infectious disease of our time?

Valerie Tweedle[1] and Robert J. Smith?[2]

[1]Department of Biology, The University of Ottawa, 585 King Edward Ave, Ottawa ON K1N 6N5 Canada; [2]Department of Mathematics and Faculty of Medicine, The University of Ottawa 585 King Edward Ave, Ottawa ON K1N 6N5, Canada

Girl baby names (Meme)



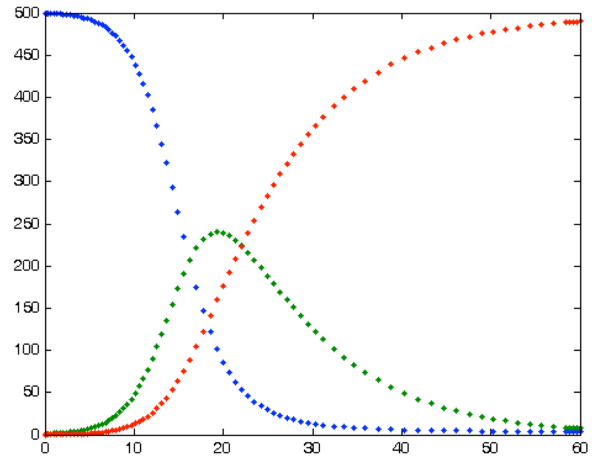
Most popular names of baby girls in each state by year

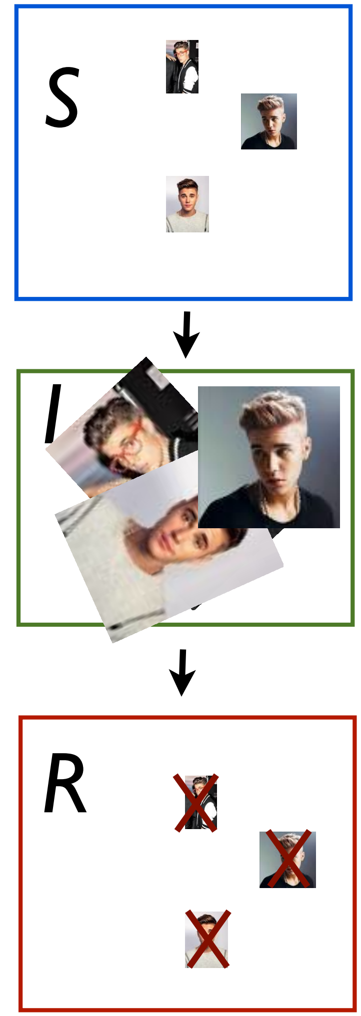
Modeling Memes

$$\frac{dS}{dt} = -\frac{\beta I S}{N}$$

$$\frac{dI}{dt} = \frac{\beta I S}{N} - \gamma I$$

$$\frac{dR}{dt} = \gamma I$$





S-I-R Model

S=Susceptible
I=Infected
R=Recovered (or removed)

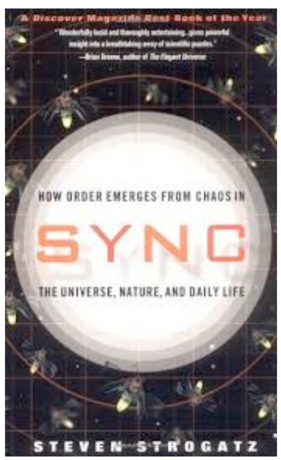
Note coupling may depend on distance or topology.

http://en.wikipedia.org/wiki/Compartmental_models_in_epidemiology

What have we learned so far . . .

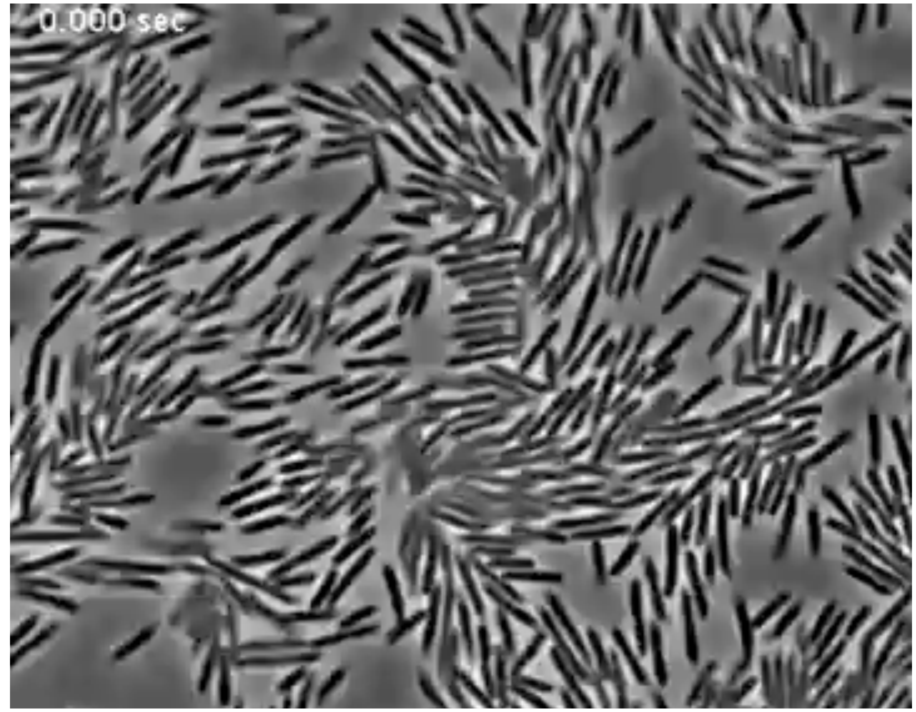
- Coupling can lead to synchronization.
- The coupling of human behavior can lead to memes.
- Humans are definitely coupled - we seem programmed to imitate each other.

What does this mean for animal behavior?



Want more? See Steve Strogatz's TED talk & book!!

Coupling of behavior leads to swarming



Swarm

(noun)

The grouping (aggregation) of coupled objects, often animals.

- Bird flocks
- Animal herds
- Schools of fish
- Insect swarms (bees, ants, locusts)

Biological groups move in a coordinated manner.

Parrish & Keshet, Nature, 1999

Groups can propagate without a leader.

Groups may have sharp edges, nearly constant density

Plate 3. Wildebeest massing in a grazing front on the Serengeti Plains. March 1973.

Sinclair, 1977

Aggregation *(noun)*

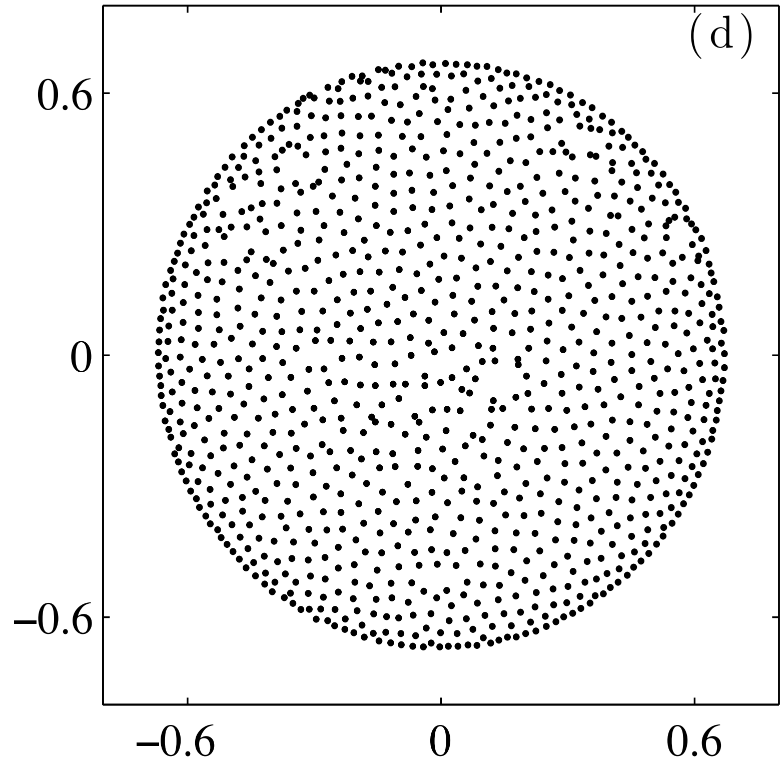
The grouping of coupled objects, often animals.

- Bird flocks
- Animal herds
- Schools of fish
- Insect swarms (bees, ants, locusts)
- Robotic swarms

Attraction/Repulsion

1. Individuals are attracted toward neighbors.

2. Individuals are repelled if they get too close.



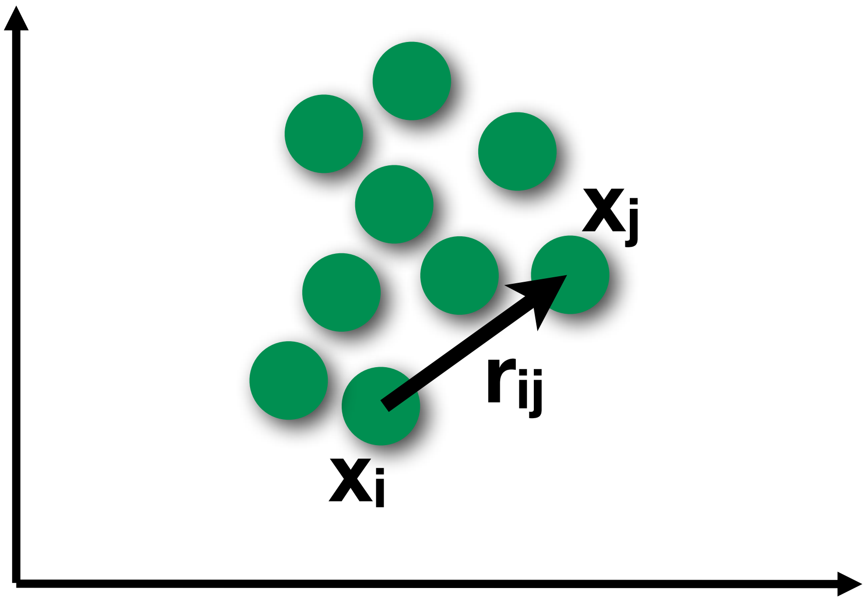
Human Swarming I

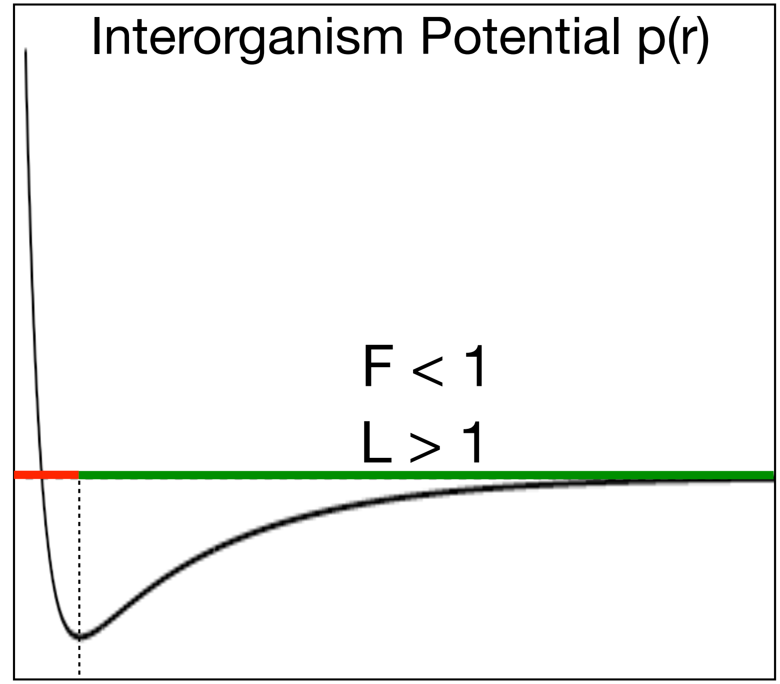
WALK slowly toward (what you perceive to be) the center of the group.

SLOW DOWN if you are within two feet of another person.

STOP if you are within one foot of another person.

Modeling Aggregations





Morse Potential

$$\dot{\vec{x}}_i = \left(\sum_{j=1}^{N} \frac{dp}{dr} \left(|\vec{r}_{ij}| \right) \frac{\vec{r}_{ij}}{|\vec{r}_{ij}|} \right)$$

pairwise additive interactions

$$p(r) = -\underline{F L e^{-r/L}} + \underline{e^{-r}}, \quad \vec{r}_{ij} = \vec{x}_j - \vec{x}_i$$

long-range attraction

short-range repulsion

Kinematic Model: Velocity = $\sum$ Social Force

Mill

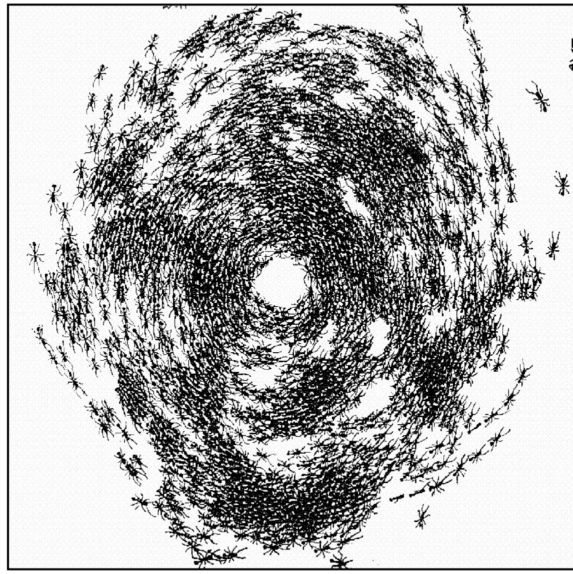
(noun)

A rotating circular swarm.

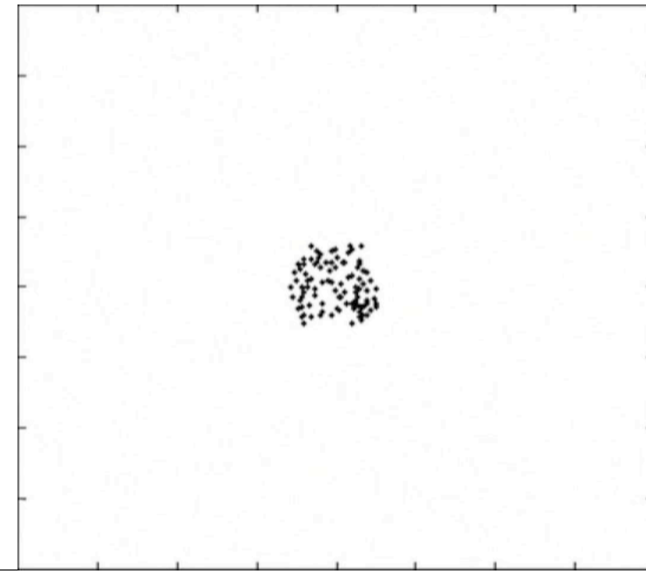
- Ants
- Fish
- Occasionally people

Individual behavior can lead to large scale pattern formation

Fish school
(Parrish & Keshet, 1999)



Ant mill
(Schnierla, 1971)



Dynamic Model with
Momentum, Attraction & Repulsion
D'Orsogna, Chuang, Bertozzi & Chayes
PRL (2006).

Dynamic Model:
Acceleration = Propulsion - Drag Forces + Social Forces

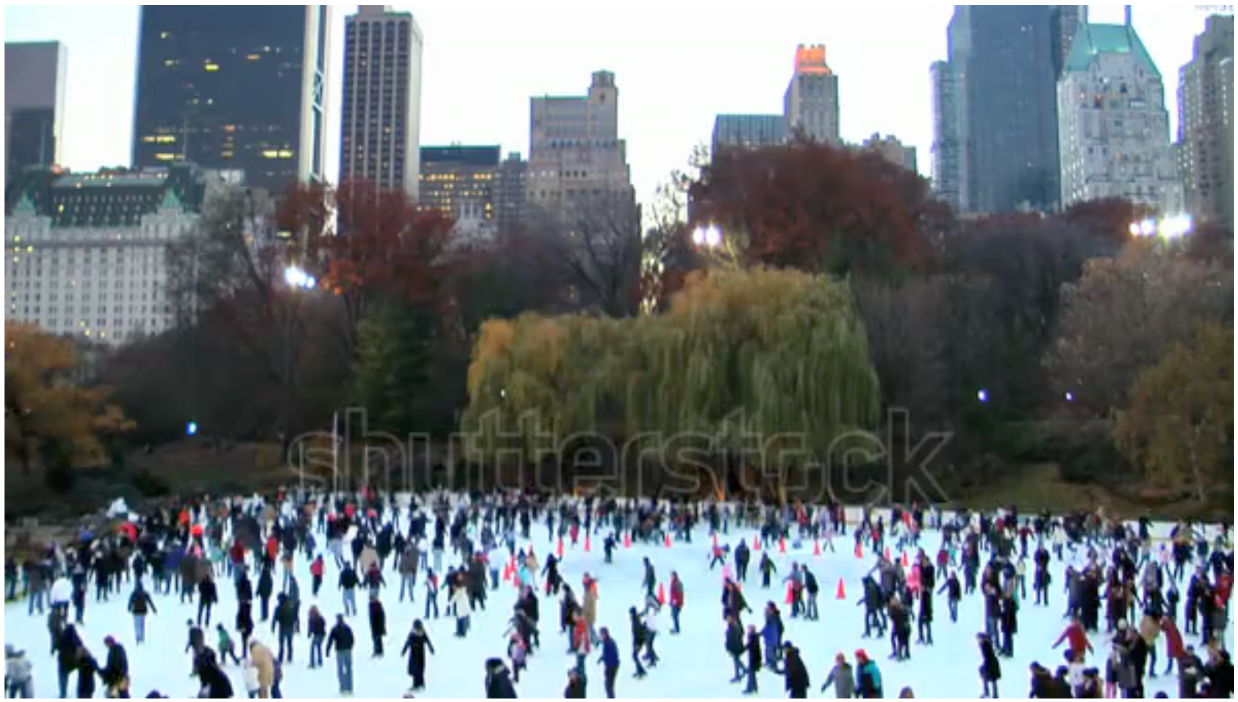
Human Swarming II

WALK at a slow, constant speed.

WALK toward the person or people you see in front of you.

TURN RIGHT if you are going to collide.

Humans can spontaneously exhibit mill behavior also

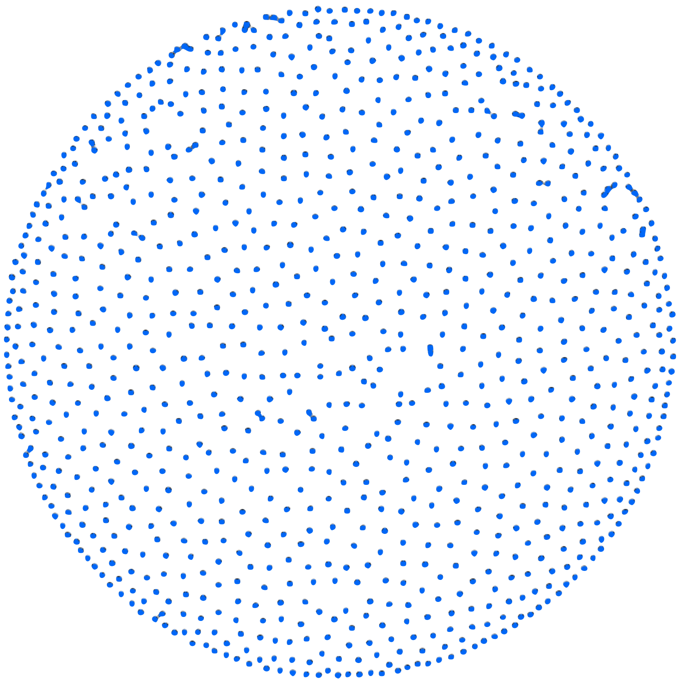


Frontiers *(noun)*

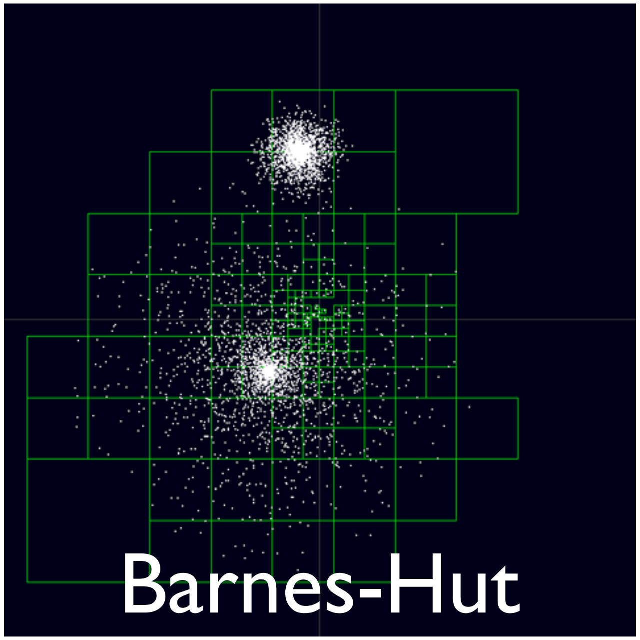
The limits of knowledge - where the action is.

Frontier #1: Efficient computation of swarms

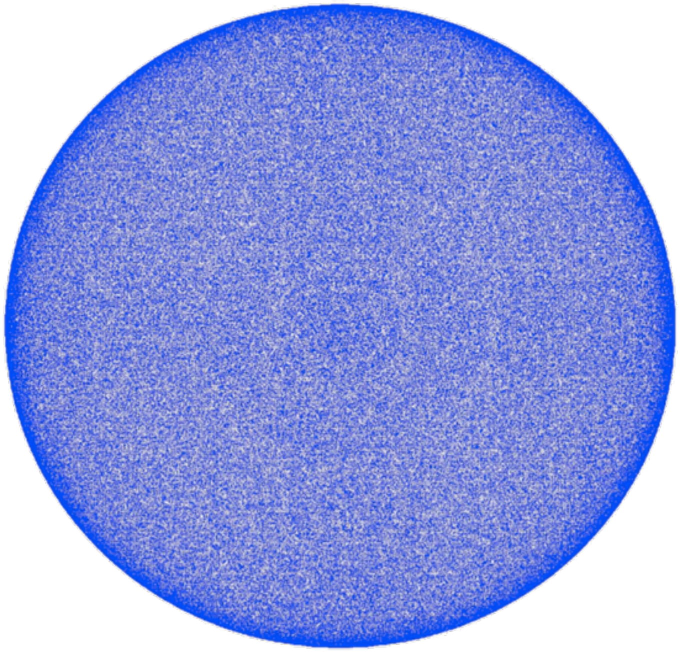
N = 1 thousand



GPU Computing



N = 1 million



Difficulty: Pairwise interactions means calculation scales like N^2

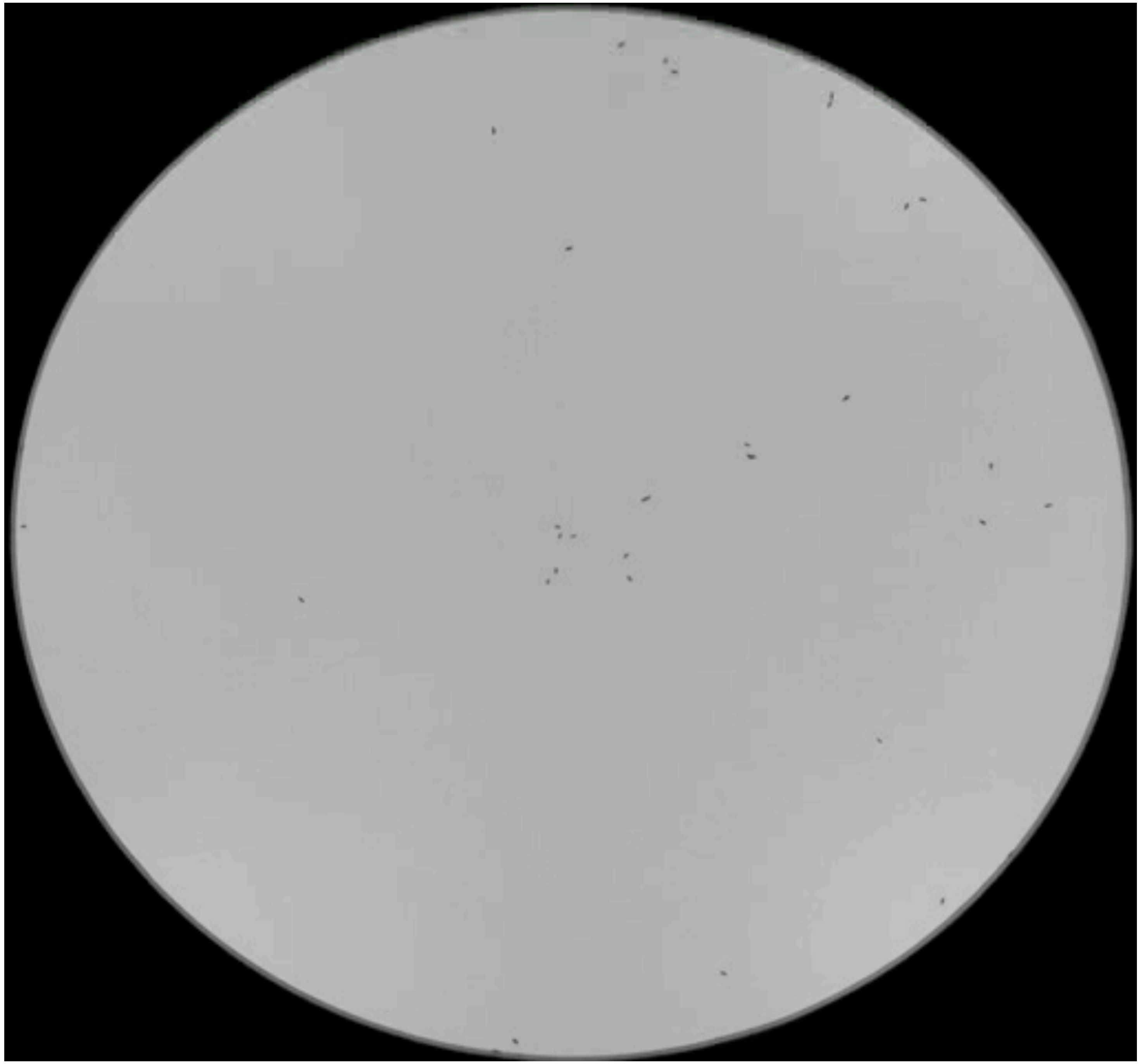
Frontier #2: Quantitative lab/field measurements

Inferring individual rules from collective behavior
Lukeman, Li & Edelstein-Keshet
(PNAS 2010)

What is a minimal model
for duck movement?

Frontier #2: Quantitative lab/field measurements

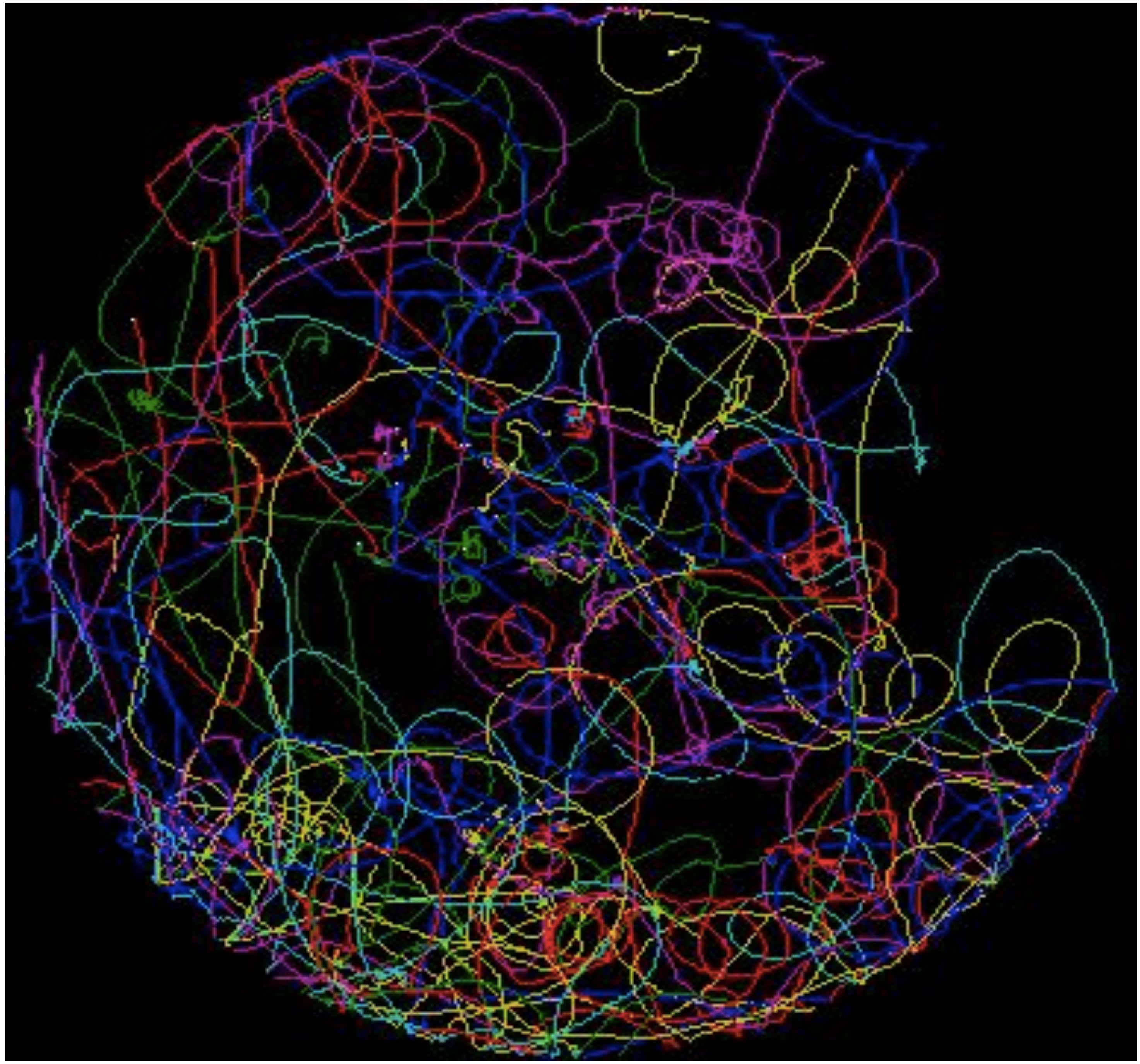
Motion tracking of pea aphids



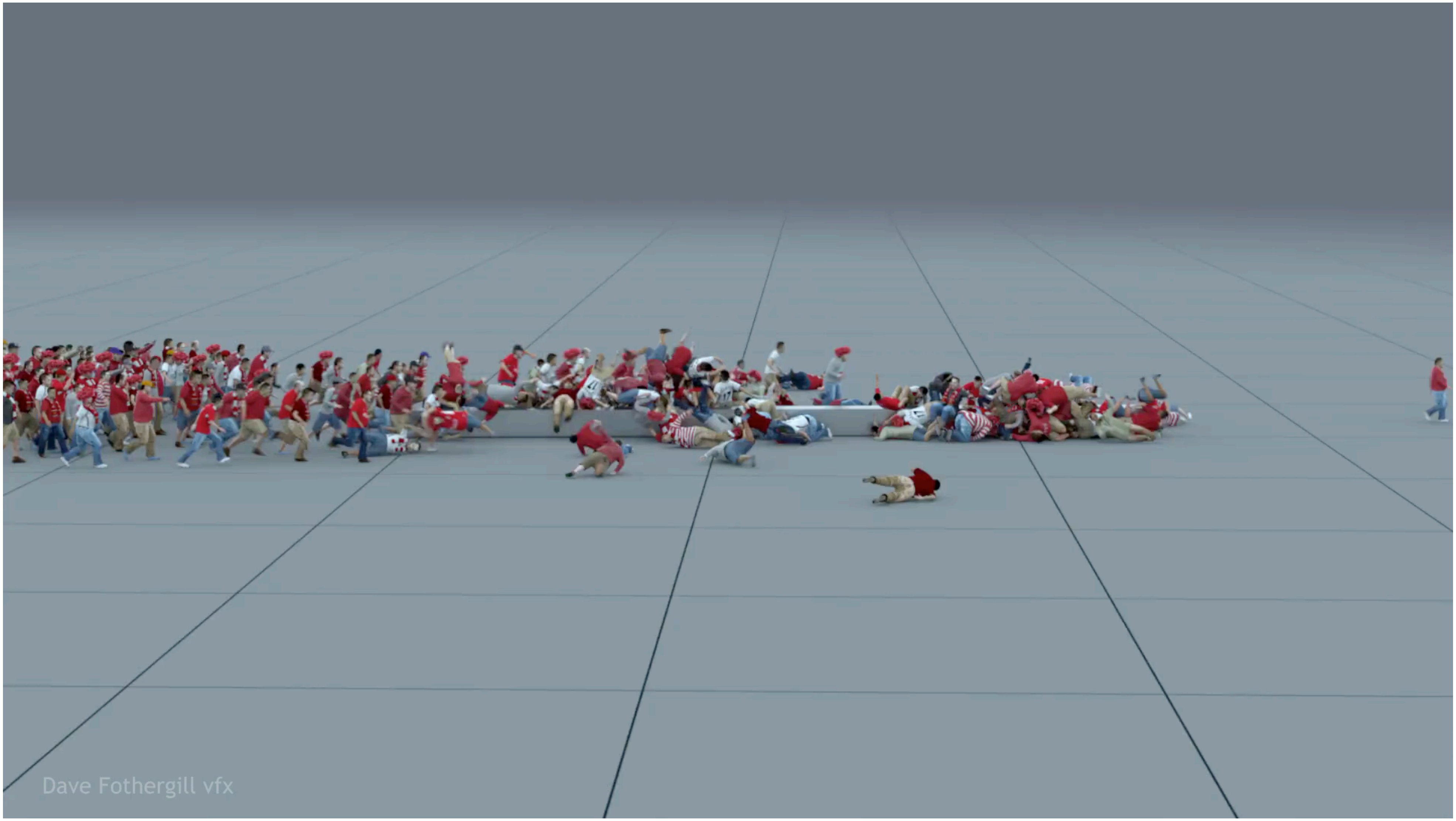
XMAC lab - Chad Topaz, Macalester College

Frontier #2: Quantitative lab/field measurements

What is a minimal model for pea aphid movement?



Frontier #3: Modeling human behavior



How well can we model human interactions?

Agent Based Modeling:
A Top Ten List

#10: Understand your goal.

9: Have a metric in mind.

8: Deal with the curse of dimensionality

7: Experimentalists are your friends

6: Think about the geometry of coupling.

5: Think laterally and borrow mercilessly.

4: Be curious and observant

3: Particles or Continuum Model?

2: Random vs. Deterministic

1: Parsimony !!