1. Use the vectors \( u = (u_1, \ldots, u_n), v = (v_1, \ldots, v_n), \) and \( w = (w_1, \ldots, w_n) \) to verify the following algebraic properties of \( \mathbb{R}^n \).
   a. \((u + v) + w = u + (v + w)\)
   b. \(k(u + v) = ku + kv\) for each scalar \( k \).

2. Use the vector \( u = (u_1, \ldots, u_n) \) to verify the following algebraic properties of \( \mathbb{R}^n \).
   a. \( u + (-u) = (-u) + u = 0 \)
   b. \(k(k'u) = (kk')u\) for all scalars \( k \) and \( k' \).

3. Consider the vectors \( u = (2, -7, 1), \ v = (-3, 0, 4), \) and \( w = (0, 5, -8) \). Compute a.) \( 3u - 4v \) and b.) \( 2u + 3v - 5w \).

4. Verify that the vectors \( u = (-1, 0, 1), \ v = (2, 4, 2), \) and \( w = (3, -3, 3) \) are orthogonal to each other.

5. Compute the Euclidean norm of the vectors \( u = (-1, 0, 1) \) and \( v = (2, 4, 2) \) and the distance between them.

6. The 1-norm of a real vector is defined by \( \|v\|_1 = \sum_{i=1}^{n} |v_i| \). Compute the 1-norm of the vectors \( u = (1, 2, 3) \) and \( v = (3, 2, 1) \).

7. The \( \infty \)-norm of a real vector is defined by \( \|v\|_\infty = \max_i |v_i| \). Compute the \( \infty \)-norm of the vectors \( u = (1, 2, 3) \) and \( v = (3, 2, 1) \).

8. Decide which of the following statements are TRUE and which are FALSE. For the TRUE ones, you have to give a proof. For the FALSE ones, you have to give a counterexample. Here we are considering \( u = (u_1, \ldots, u_n), \ v = (v_1, \ldots, v_n), \) and \( [u, v] = (u_1, \ldots, u_n, v_1, \ldots, v_n) \) is the vector formed by concatenating them.
   a. \( \|u\|_1 + \|v\|_1 = \|[u, v]\|_1 \)
   b. \( \|u\|^2_2 + \|v\|^2_2 = \|[u, v]\|^2_2 \)
   c. \( \|u\|_\infty + \|v\|_\infty = \|[u, v]\|_\infty \)

9. Compute the following:
   \[
   4 \begin{pmatrix} 1 & 2 \\ 4 & 6 \end{pmatrix} - \begin{pmatrix} 1 & 0 \\ -1 & 0 \end{pmatrix} - 2 \begin{pmatrix} 5 & 1 \\ 4 & 3 \end{pmatrix}.
   \]

10. Find the values of \( x, y, z, \) and \( w \) so that the following matrix equality holds:
    \[
    3 \begin{pmatrix} x & y \\ z & w \end{pmatrix} = \begin{pmatrix} x & 5 \\ -1 & 2w \end{pmatrix} + \begin{pmatrix} 4 & x + y \\ z + w & 3 \end{pmatrix}.
    \]