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Conference on Hyperbolic Conservation Laws and Continuum Mechanics

In Honor of Constantine Dafermos' 70th Birthday

Abstracts

Stuart S. Antman
University of Maryland

Quasilinear Systems of Visco-Elasticity

The partial differential equations governing strain-rate viscoelasticity form quasilinear parabolic-hyperbolic systems. This lecture describes two classes of problems for such systems. The first class of problems is to determine the nature of invariant dissipative mechanisms for the quasilinear hyperbolic equations of nonlinear elasticity and to determine how such dissipative mechanisms affect computation and shock structure. The second class of problems is to justify asymptotic analyses of the equations for heavily burdened viscoelastic bodies. The formal analyses of such problems are reduced to the study of elliptic systems of equations, parabolic systems of equations, and curious systems of ordinary differential equations.

Claude Bardos
University of Paris Diderot

About Incompressible Euler limit of Solutions of Navier-Stokes and Boltzmann Equation in the Presence of Boundary Effects

This is a report on ongoing joint work with François Golsea and Lionel Paillard. It is well known that in any dimension (even in 2d the limit (when the Reynold number goes to) is in the presence of boundary, a challenging open problem.

Results are simpler when the fluid satisfies a Navier boundary condition and the problem is completely open when the fluid satisfies for finite Reynold number a Dirichlet boundary condition. The only general (always valid) mathematical result being a classical theorem of Tosio Kato.

In the incompressible finite Reynold number limit the solution of the Boltzmann equation (with boundary and accommodation effect) converges to a Leray solution of the Navier Stokes equation with a Navier Boundary condition which depends on the accommodation coefficient (Kazuo Aoki… Nader Masmoudi and Laure Saint Raymond).

On the other hand it has been observed by several researchers from Yoshio Sone to Laure Saint Raymond that with convenient scalings (infinite Reynolds number) and in the absence of boundary the Boltzmann equation leads to the incompressible Euler equation.

Hence we try to adapt to this limit, in presence of boundary with accommodation, what is known or conjectured at the level of the Navier Stokes limit.

Stefano Bianchini
SISSA

SBV Regularity for General Hyperbolic System of Conservation Laws

Alberto Bressan
Penn State University

Non-classical Control Problems and Stackelberg Feedback Equilibria

When the equation describing the evolution of a system contains the spatial gradient of a feedback control, entirely new phenomena arise. The talk will review the concept of Stackelberg equilibrium for differential games, which yield "non-classical" problems of optimal feedback control. Examples show that these problems can lead to a rich variety of linear and nonlinear PDEs. Some results on the possible reduction to more familiar problems of optimal control will be stated.

Gui-Qiang G. Chen
University of Oxford

Multidimensional Discontinuities and Free Boundary Problems in Hyperbolic Conservation Laws

In this talk we will present several free boundary problems for the stability of multidimensional discontinuities and the existence of fundamental wave patterns in hyperbolic conservation laws.
The discontinuities include shock waves, vortex sheets, and entropy waves. Some recent developments will be reviewed and discussed.

Further trends, perspectives, and open problems in this direction will be also addressed.

Rustum Choksi
McGill University

Asymptotic Regions and Global Minimization for a Variational Problem with Long-Range Interactions

Energy-driven pattern formation induced by competing short and long-range interactions is common in many physical systems. This talk will address a variational problem which may be viewed as a mathematical paradigm for periodic pattern formation induced by these energetic competitions. I will focus on rigorous results pertaining to global minimization in certain asymptotic regimes. This is recent work with M. Peletier (TU Eindhoven) and work with G. Alberti (Pisa) and F. Otto (MPI, Leipzig).

I will also show numerical simulations (work with J.F. Williams and M. Maras at Simon Fraser) in two and three space dimensions which attempt to access the ground state.

Camillo De Lellis
University of Zurich

Oscillations and Admissible Solutions of Hyperbolic Systems of Conservation Laws

In this talk I will review the results of some recent joint papers with László Székelyhidi, where we prove, among other things, the existence of infinitely many admissible solutions to the Cauchy problem for the *p*-system in 2 space dimensions. I will show how this is linked to some "classical" considerations about oscillations and partial differential constraints, in the spirit of DiPerna and Tartar. I will also discuss the crucial role played by the regularity of the initial data.

Hermano Frid
IMPA, Brazil

Remarks on the Theory of the (Extended) Divergence-measure Fields

In this talk we will review the basic results about the -valued measures whose distributional divergence is also a measure, the so called (extended) divergence-measure fields as introduced by Chen and Frid (2003) with improvements made by Silhavy (2008). We will also discuss its main application to the Euler equations in gas dynamics.

Barbara Keyfitz
The Ohio State University

A New Look at Singular Shocks

Some examples of singular shocks, which are solutions of conservation laws of very low regularity, were found by Herbert Kranzer and the speaker in the 1980's, and a theory was developed by Michael Sever that encompassed the examples, and categorized the differences between singular shocks and delta shocks, a similar type of low-regularity solution. However, the topic has remained merely a curiosity.

Recently, some new models and experiments in chromatography suggest that these somewhat pathological waves may have a physical meaning, and that the examples studied so far may not cover the entire spectrum of singular behavior of solutions of conservation laws. This talk will summarize what is known, and what might be conjectured about data and singular shocks.

Tai-Ping Liu
Academia Sinica

Conservation Laws with Physical Viscosity

With Yanni Zeng, we study the propagation of waves over a shock profile for system of conservation laws with physical viscosity. Navier-Stokes equations in gas dynamics, the magnetohydrodynamics, and full nonlinear elasticity equations are important examples. We use the Green's function approach. There are rich nonlinear wave phenomena as a consequence of the wave coupling and the compressive nature of the shock waves.

Andrew J. Majda
Courant Institute of Mathematical Sciences

Mathematical Strategies for Real Time Filtering of Turbulent Dynamical Systems

An important emerging scientific issue in many practical problems ranging from climate and weather prediction to biological science involves the real time filtering and prediction through partial observations of noisy turbulent signals for complex dynamical systems with many degrees of freedom as well as the statistical accuracy of various strategies to cope with the "curse of dimensions". The speaker and his collaborators, Harlim (North Carolina State University), Gershgorin (CIMS Post doc), and Grote (University of Basel) have developed a systematic applied mathematics perspective on all of these issues. One part of these ideas blends classical stability analysis for PDE's and their finite difference approximations, suitable versions of Kalman filtering, and stochastic models from turbulence theory to deal with the large model errors in realistic systems. Many new mathematical phenomena occur. Another aspect involves the development of test suites of statistically exactly solvable models and new NEKF algorithms for filtering and prediction for slow-fast system, moist convection, and turbulent tracers. Here a stringent suite of test models for filtering and stochastic parameter estimation is developed based on NEKF algorithms in order to systematically correct both multiplicative and additive bias in an imperfect model. As briefly described in the talk, there are both significantly increased filtering and predictive skill through the NEKF stochastic parameter estimation algorithms provided that these are guided by mathematical theory. The recent paper by Majda et al (Discrete and Cont. Dyn. Systems, 2010, Vol. 2, 441-486) as well as a forthcoming introductory graduate text by Majda and Harlim (Cambridge U. Press) provide an overview of this research.

Benoit Perthame
Université Pierre et Marie Curie

Darwinian Evolution and the Constrained Hamilton-Jacobi Equation

Living systems are subject to constant evolution through the three processes of population growth, selection and mutations, a principle destablished by C. Darwin. In a very simple, general and idealized description, their environment can be considered as a nutrient shared by all the population. This allows certain individuals, characterized by a 'phenotypical trait', to expand faster because they are better adapted to use the environment. This leads to select the 'fittest trait' in the population (singular point of the system). On the other hand, the new-born individuals undergo small variations of the trait under the effect of genetic mutations. In these circumstances, is it possible to describe the dynamical evolution of the current trait? A new area of population biology that aims at describing mathematically these processes is born in the 1980's under the name of 'adaptive dynamics' and, compared to population genetics, considers usually asexual reproduction, a continuous phenotypical trait and population growth.

We will give a self-contained mathematical model of such dynamics, based on parabolic equations, and show that an asymptotic method allows us to formalize precisely the concepts of monomorphic or polymorphic population. Then, we can describe the evolution of the 'fittest trait' and eventually to compute various forms of branching points which represent the cohabitation of two different populations.

The concepts are based on the asymptotic analysis of the above mentioned parabolic equations once appropriately rescaled. This leads to concentrations of the solutions and the difficulty is to evaluate the weight and position of the moving Dirac masses that describe the population. We will show that a new type of Hamilton-Jacobi equation, with constraints, naturally describes this asymptotic. Some additional theoretical questions as uniqueness for the limiting H.-J. equation will also be addressed.

This talk is based on collaborations with G. Barles, J. Carrillo, S. Cuadrado, O. Diekmann, M. Gauduchon, S. Genieys, and P.-E. Jabin, S. Mirahimmi, S. Mischler and P. E. Souganidis.

Tommaso Ruggeri
University of Bologna

Can Constitutive Relations be Represented by Non-local Equations?

The modern theory of extended thermodynamics, shows that the popular “constitutive” equations of continuum mechanics of non-local form are in reality approximations of balance laws, when some relaxation times are neglected. We recall, for example, the Fourier's equation, the Navier-Stokes' equations, the Fick's equation, the Darcy's law and several others. This idea suggests that the “authentic” type of constitutive equations are local and, therefore, the differential systems of mathematical physics are hyperbolic rather than parabolic. Another consequence is that these equations do not need to satisfy the so called objectivity principle that on the contrary still continues to be valid only for the "authentic" constitutive equations. However these limit non-local equations are useful not only because they are used in normal physical situations but also because they permit to obtain the evaluation of non-observable quantities such as the velocity or the temperature of each constituent of a mixture. Considerations are also made with regard to the formal limits from hyperbolic versus parabolic and from hyperbolic versus hyperbolic, between a system and a subsystem. We discuss the main analytical properties with respect to the global existence of smooth solutions for dissipative hyperbolic systems.

Denis Serre
ENS de Lyon

The Numerical Measure of a Complex Matrix. Lacunas of Hyperbolic Differential Operators

According to the Toeplitz-Hausdorff Theorem, the numerical range of an $N \times N$ matrix is a convex compact subset of the complex plane. In this joint work with Thierry Gallay, we introduce the notion of 'numerical measure'. This is a probability measure, supported by the numerical range. it enjoys amazing properties. For normal matrices, the measure density is a bivariate Box-spline with nodes at the eigenvalues. The situation is more complex for non-normal matrices, but the density can be reconstructed explicitly by means of the inverse Radon transform (retro-projection formula). Its singular support is an algebraic planar curve Σ with cusps; if $z \notin \Sigma$, there are N tangents from z to Σ. Connected components of the complement of Σ, in which all these tangents are real, turn out to be lacunas of a first-order hyperbolic differential operator in $2+1$ variables. All such operators can be treated that way. These lacunas can be characterized as zones where the numerical density is polynomial of degree at most $N-2$.

Marshall Slemrod
University of Wisconsin

Entropy and Isometric Embedding

Costas Dafermos has championed the role of entropy as a key component in understanding systems of conservation laws arising in continuum mechanics. Following in his footsteps I will try to show how similar ideas are also useful in isometric embedding of Riemannian manifolds.

Panagiotis Souganidis
University of Texas at Austin

Scalar Conservation Laws with "Bad" Time Dependence

Konstantina Trivisa
University of Maryland

Dissipation, Diffusion & Entropy in the Balance Laws of Continuum Physics

In this talk the concepts of dissipation, diffusion and entropy will be discussed in connection with specific models arising in Continuum Physics.

Athanasios E. Tzavaras
University of Crete

On the Structure and Properties of the Equations of Nonlinear Elasticity

The equations of elastodynamics are a paradigm of a system of conservation laws where the lack of uniform convexity of the stored energy function poses challenges in the mathematical theory. Nevertheless, the existence of certain nonlinear transport equations for null-Lagrangeans reinforces the efficacy of the entropy as a stabilizing factor and recovers the strength associated with uniformly convex entropies in hyperbolic systems. It turns out that elastodynamics with polyconvex stored energy can be embedded into a larger symmetric hyperbolic system and visualized as constrained evolution leading to a variational approximation scheme and an existence theory for measure valued solutions satisfying certain kinematic constraints in the weak sense. It provides a framework, in conjunction with the relative entropy method, to establish convergence of viscosity approximations or convergence of time-step approximants to smooth solutions of polyconvex elastodynamics. In addition, when a smooth solution is present it is unique within the class of measure valued solutions.

The system of radial elastodynamics for isotropic elastic materials is an interesting case study. We present the format of the enlarged system in this special case with the objective of assigning a mechanical interpretation in the nonlinear transport constraints. It turns out that one can construct via variational approximation solutions that obeys the impenetrability of matter constraint.

Shih-Hsien Yu
University of Singapore

Construction of Green's Functions for the Boltzmann Equations

There are been many progresses in the connection between the Boltzmann equations and fluid mechanics through the detailed wave propagation structures in the solutions of the Boltzmann equation. The basic component in this approach is the Green's function with exponentially constructed sharp pointwise structure so that the Green's function is refined enough to study the nonlinear wave interactions

In this talk, we will review the constructions of the Green's functions of the Boltzmann equation linearized around various wave patterns such as boundary layer, shock layer, etc. In the construction, different physical characteristics such as the macro-micro decomposition, the long wave-short wave decompositions, particle-like and wave-like dual property, T-C scheme in the consideration of conservation laws, ..., are binded together to give a global structure of the Green's function.