Solving the binary black hole problem (again and again and again....)

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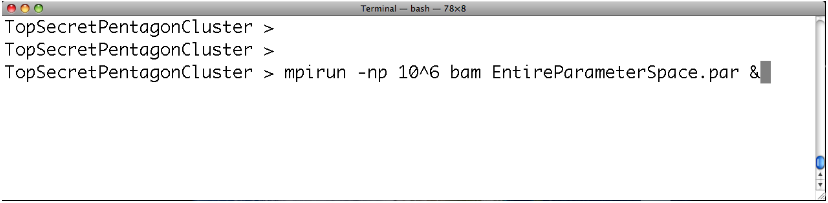
Cardiff University



ACCGR Workshop
Brown University, May 21 2011

Work in collaboration with...

- **Parameswaran Ajith, Caltech**
- **Céline Cattoën, Alberta**
- **Sascha Husa, UIB**
- **Frank Ohme, AEI Potsdam**
- **Michael Pürrer, Vienna**
- **Patricia Schmidt, Cardiff**

Science from GW observations of BH mergers requires accurate theoretical waveforms across the entire parameter space

- **The parameter space is large**
- **Modeling generic systems is poorly understood**
- **NR simulations are computationally expensive**
- **Length and accuracy requirements are unclear**
- **Even setting up the simulations is difficult...**

Problems from the outset: initial parameters

We need to determine initial parameters for low-eccentricity inspiral

Iterative procedures have been proposed

- works with quasi-equilibrium conformal-thin-sandwich excision data (SpEC)

[Pfeiffer, *et. al.*, 2007; Mroue, *et. al.*, 2010; Buonanno, *et. al.*, 2010]

For puncture data, we have used PN-based methods

- First guess: use PN prediction for circular orbits (no inspiral). $e \sim 10^{-2}$
  [Brügmann, *et. al.*, PRD 77 024027 (2008)]

- Improvement: integrate full PN equations for inspiral. $e \sim 10^{-3}$
  Husa, MDH, *et. al.*, PRD 77, 044037 (2008)]

Reducing eccentricity further

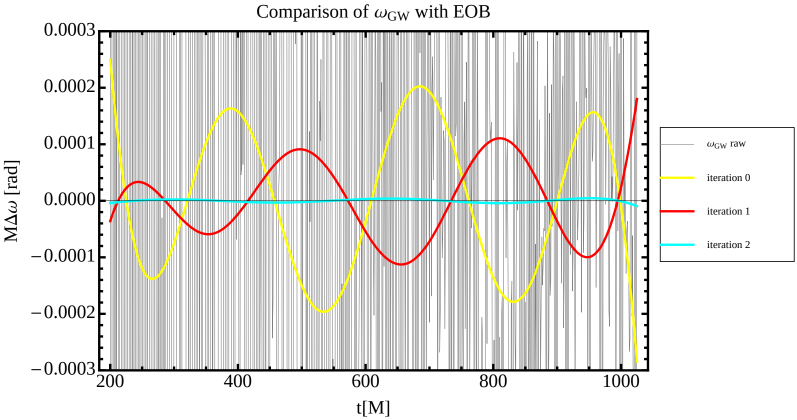
It is difficult to improve on $e \sim 0.005$ with a naive iterative procedure

- Numerical noise in the waveforms
- Gauge adjustments over the first orbit
- Long-term gauge effects in the puncture motion

A cleaner eccentricity measure can be made from the (filtered) GW phase

An iterative procedure can be based on full PN information

Reducing eccentricity further

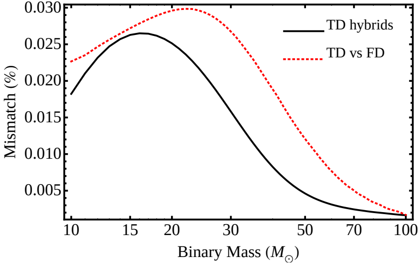


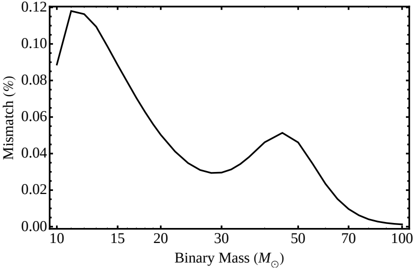
See poster by Michael Pürrer

How long do numerical waveforms need to be?

We can produce complete IMR waveforms by making PN + NR hybrids

- PN errors dominate the errors in these hybrids
- The longer the NR waveform, the better the hybrid



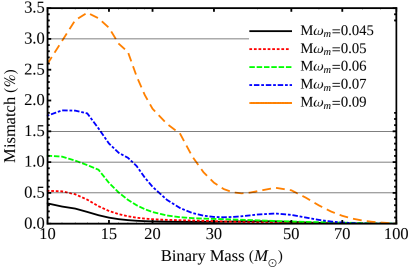


Without PN error estimates, how do we decide NR waveform lengths?

First attempt: make hybrids with several approximants and compare

MDH, Husa, Ohme, Ajith, PRD 82 (2010) 124052]

$q = 1$ nonspinning



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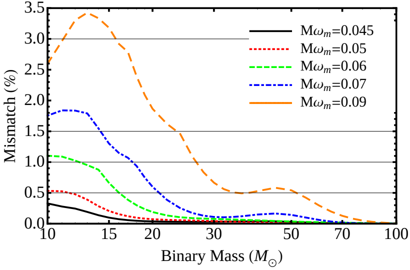
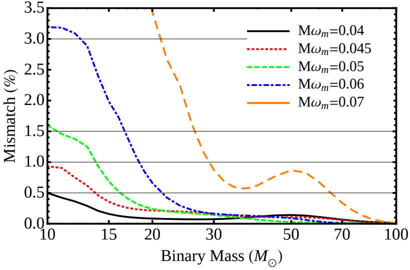
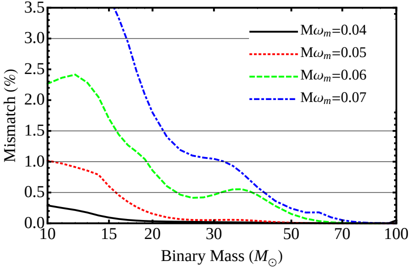


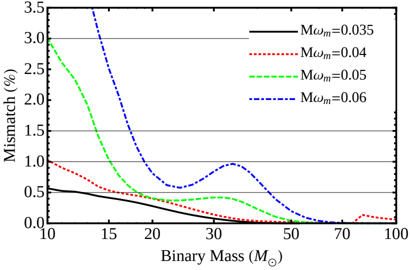
$q = 4$ nonspinning



$q = 1$, $\chi_1 = \chi_2 = -0.5$



$q = 1$, $\chi_1 = \chi_2 = +0.5$



Length requirements

In a search, optimize match calculation over all parameters (with only one NR waveform, can only optimize over mass)

But, we can compute matches *without* NR waveforms.

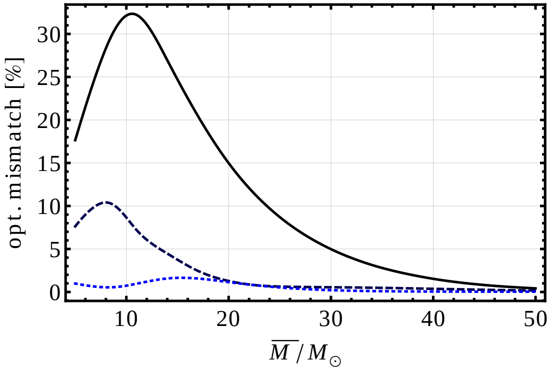
The fully optimized match depends on only

- the match between the two PN approximants
- the phase difference of the approximants at the matching frequency
- the ratio of the total power in the PN and NR parts
- the accumulated phase difference of the PN parts (when considering different parameters)

All of these are integrated quantities, that can be accurately modeled already by phenomenological waveforms!

Length requirements

$q = 4$, nonspinning



- Nonspinning cases: < 10 orbits are acceptable up to $q = 10$
- Spinning cases: not so good
  — *but* higher order spin terms may improve the PN accuracy

Parameter estimation

Parameter estimation errors depend on signal strength *and* waveform errors

The waveform errors are irrelevant if the waveforms are "indistinguishable"

- Several studies have considered length requirements for indistinguishability
- **Bad news:** we need 100s or 1000s of NR orbits

[Damour, *et. al.* (2010); MacDonald, *et. al.* (2010); Boyle (2011)]

Obvious conclusion: we need longer waveforms

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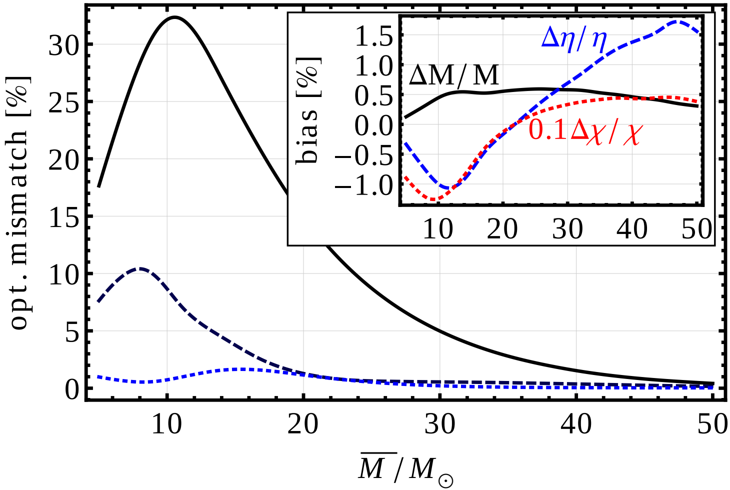
Obvious conclusion: we need longer waveforms

Too bad: we won't have those waveforms by 2015!

The real question is:
what will be the errors from waveforms with ~ 10 cycles?

Parameter bias with hybrids

($q = 4$ nonspinning)



Waveform models

The spinning-binary parameter space is vast:

- Binaries are characterized by
  - the mass ratio $\eta = m_1 m_2/(m_1 + m_2)^2$. (1 parameter)
  - the spins $\mathbf{S}_1$ and $\mathbf{S}_2$ of each BH. (6 parameters)
- *Assume* we find a model with a *cubic* dependence on the parameters
- Then we need *at least* $4^7 \approx 16,000$ simulations!
- *And* the physics now includes precession of the black-hole spins, and of the binary's orbital plane...

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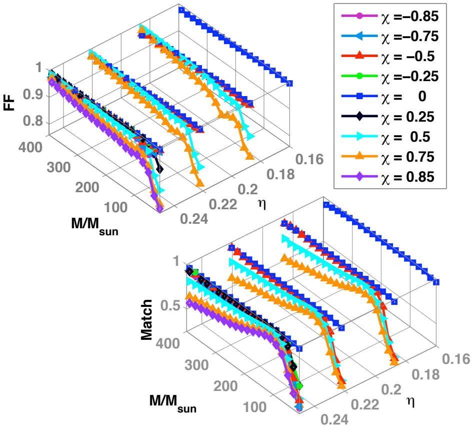
However...

- As a first approximation, describing non-precessing binaries may be enough
- *Now we have to deal with only three parameters, η, S_1, S_2.*
- Plus: the waveforms vary most strongly with the *total* spin, χ.
- *Now we have only two parameters, η, χ.*

Features of the new spinning-binary model

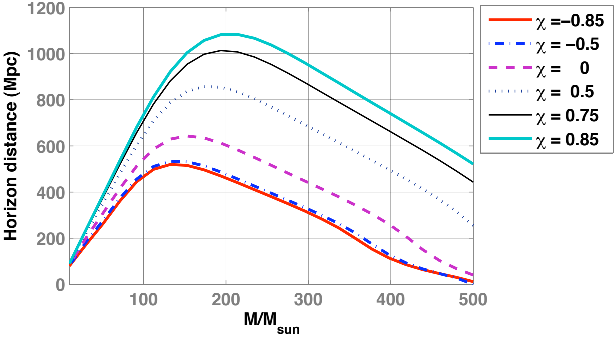
What would we lose if we used only *nonspinning waveforms* **in searches?**



- FF < 0.8 suggests that over 50% of high-spin binaries may not be detected!
- Match ≈ 0.3 means that parameters will be significantly biased.

Features of the new spinning-binary model

Can we see any further with these new waveforms?



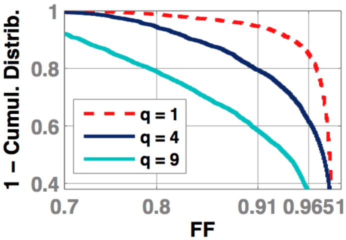
We may see high-spin binaries twice as far away as nonspinning binaries

This is excellent news: most astrophysical BHs may be highly spinning!

Can this simple model also detect generic binaries?

A preliminary study:

- We do not have generic merger waveforms
- We *can* produce generic PN waveforms
- Perform a monte-carlo comparison of generic against non-precessing waveforms
- With matches above 0.965 we have:
  - 85% at $q = 1$
  - 62% at $q = 4$
  - 37% at $q = 9$.



We find that

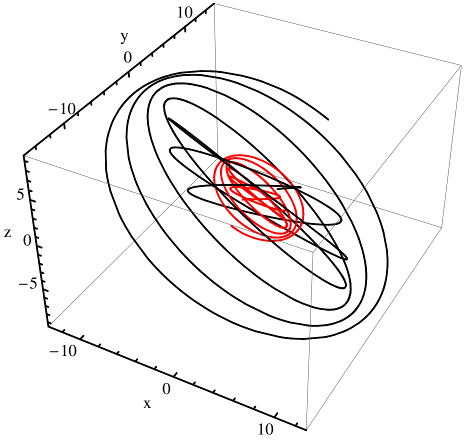
- For comparable mass binaries, the non-precessing model finds most signals
- For larger mass ratios, a full generic model is needed

[Ajith, MDH, Husa, *et al*, arXiv:0909.2867, PRL, in press]

Eventually we have to face up to generic spins

If the spins are not (anti-)aligned with the orbital angular momentum

- The spin directions will evolve (precess)
- The orbital plane will also precess
- The resulting dynamics are complex



Consider

- $q = 3$
- $\chi_1 = 0$, $\chi_2 = 0.75$
- $\mathbf{S} \cdot \mathbf{L} = 0$
- **A maximally precessing case**

How can we possibly model such systems?

Precession of the orbital plane mixes up the mode structure

- The quadrupole mode is no longer clearly dominant
- Energy is distributed across all modes.
- A simple phenomenological ansatz looks to be out of the question...

BUT...

- Rotate frame of reference to maximize $\Psi_{4,22}$.
- Maximization is accurate to within fractions of a radian
- Locates direction of the orbital angular momentum
- Defines a unique "quadrupole aligned" waveform

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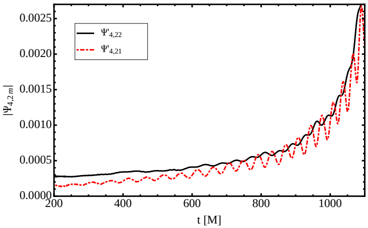
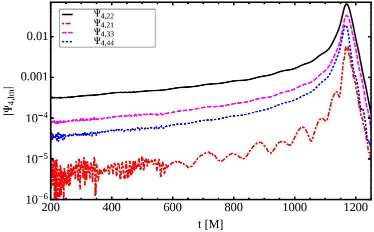
BUT...

It *is* possible to disentangle the complex motion
And the mode structure can be made simpler



$\Rightarrow$

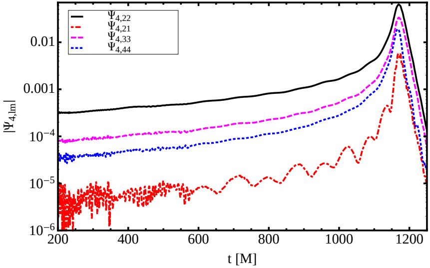


How can we possibly model such systems?

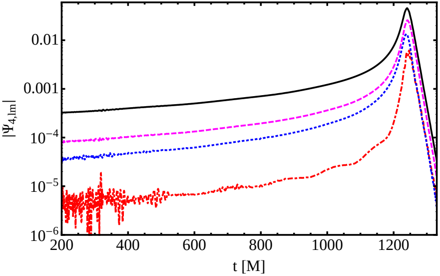
Non-precessing mode structure is retained

- $q = 3$ with $\mathbf{S} \cdot \mathbf{L} = 0$ comparable to $q = 3$ nonspinning
- Waveform modeling may now be simplified

$q = 3$, $\mathbf{S} \cdot \mathbf{L} = 0$



$q = 3$ nonspinning



- How well does this extend to more general cases?
- How well do QA and non-precessing waveforms agree?

See the poster by Patricia Schmidt [Schmidt, *et. al.*, arXiv:1012.2879]

Spectral element methods (SEM)

(With Céline Cattoën)

- First test case: stationary 1+log trumpet solution of Schwarzschild
- Ideal test case: can treat each variable separately

Preliminary results:
Look at A_{xx}, which is badly discontinuous at the puncture.

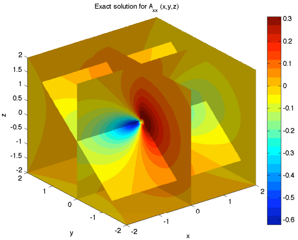
Near the puncture,

$$A_{xx} = A\frac{-2x^2 + y^2 + z^2}{r^2},$$

so

$$A_{xx}|_{y=z=0} = -2A$$

$$A_{xx}|_{x=y=0} = A.$$



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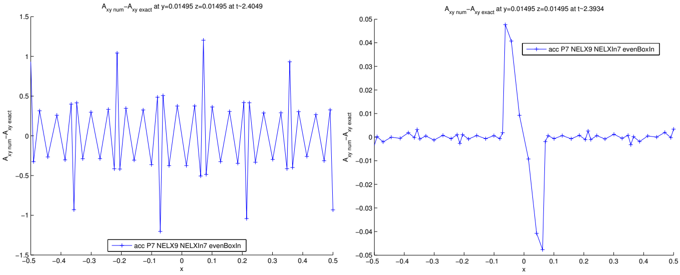
A_{xx} near the puncture, with and without filtering:



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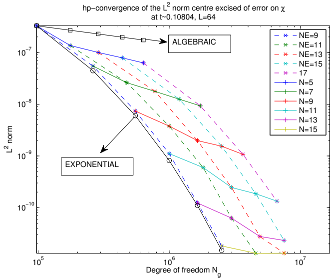
Exponential convergence is maintained away from the puncture



Conclusions

By the time Advanced LIGO is operational

we won't have perfect waveform models

but *acceptable* **models are feasible**

How to produce those models

remains an open question